Economic vulnerability of southern US slash pine forests to climate change
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A B S T R A C T

It is widely accepted that pine plantation forests will play a critical role in climate change (CC) mitigation,
but their vulnerability to CC impacts raises questions about their role. We modeled the impacts of changing climatic variables on forest growth, optimal harvest age, and land expectation value (LEV) for 11 representative slash pine sites in the Southeastern US, under two alternative climate scenarios (RCP4.5 and 8.5). Our coupled modeling approach incorporated the 3-PG biological process model, a generalized carbon sequestration economic model, and Pressler’s indicator rate formula to determine relative changes in prices, timber and carbon production. We generally found weak impacts of CC on slash pine LEVs and optimal harvest ages, but our results were sensitive to site productivity and location. CC increased LEVs in sites with low productivity for both RCPs. While a 1 ◦C increase led to the greatest LEV increase in Northeastern sites with low and moderate forest productivity conditions, Southeastern sites showed the greatest decreases in LEV. Higher (lower) future land values would shorten (lengthen) the current harvest age for slash pine. Changes in the rate of carbon and stumpage prices had the greatest impact on the rate of marginal economic revenues of slash pine.
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Introduction

There is broad consensus in the scientific community that climate change (CC) will continue to worsen without stringent policy interventions (Moore and Diaz, 2015). Recent projections (e.g., IPCC, 2013a) suggest that global temperatures and concomitant weather impacts will increase by a considerable amount (>+4.8 ◦C by 2100), but with high spatiotemporal variation (Collins et al., 2013). For example, by 2100 the southern United States (US) is expected to see up to +3 ◦C increase in mean temperatures (Kirtman et al., 2013), and +10% to 20% increase in mean precipitation in winter months (IPCC, 2013a).

We know that as forests grow, they capture atmospheric carbon dioxide (CO2) and store it as biomass. For example, in the heavily-forested southeastern US, Han et al. (2007) predict that nearly 1/4th of the region’s GHG emissions – a considerable amount – could be offset by forests. We also know that increasing forest biomass and extending the forest estate are relatively cost-effective CC mitigation approaches (Couture and Reynaud, 2011; Gren and Carlsson, 2013), and for that reason afforestation and reforestation are central to several greenhouse gas (GHG) emissions reduction programs and policies (e.g., EPA’s Clean Power Plan (Soto et al., 2016), and the Kyoto Protocol to the United Nations Framework Convention on Climate Change (UNFCC, 1998).

From the perspective of ecological and biological processes, we have a strong and improving understanding of the relationship between forests, the carbon cycle, and climate change. It is clear that climate change will impact the structure and function of forest ecosystems (Johnsen et al., 2014), though the direction and magnitude of these impacts is thought to vary considerably across the spatiotemporal gradient, and this is an area of active debate. While higher concentration of atmospheric carbon dioxide (CO2) and temperatures are expected to increase forest pine growth in the Southern US (Wertin et al., 2012), this increase in productivity may be hampered by drought conditions and water stress (Wertin et al., 2010). CC is also associated with an increase in forest disturbance risks, which may offset any productivity gains. Wildfire frequency and intensity are expected to increase with projected increases in temperatures in the region (Liu

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et al., 2014; Susaeta et al., 2016a). Pest outbreaks, hurricanes and invasive species are also expected to increase with CC, thus affecting ecological and productive functions of forests (Haim et al., 2011; Susaeta et al., 2016b). In this way, CC can have significant impacts on this important carbon sequestration role1 that southern forests could play in the US.

Considerably less attention has focused on the economic feasibility of this approach. Throughout much of the world, private forest ownership is the dominant form of forest land tenure. Indeed, a major threat to forests and forest C stocks is land use change. Forest carbon stocks are actually projected to fall from 2030 to 2060 due to losses in forest areas (Hugget et al., 2013).

Robust markets for forest products (e.g., timber and bioenergy) and services (e.g., C sequestration) can counter this effect, and are important policy considerations. Forests play an important role in the economic system, meeting society’s needs for raw materials, mostly timber and fiber. The southern US is one of the most productive tree-growing areas in the world. Southern forests occupy 87 million hectares (ha), provide 12% of the world’s industrial roundwood, 19% of the world’s pulp and paper products; they also provide 53% of the sawlog and veneer products in the US, and 72% of the pulpwood in the US (Smith et al., 2009). Depending on assumptions about forest product demand and productivity of forest plantations, southern US forests are forecast to increase total timber production by 25–70% between 2010 and 2060 (Hugget et al., 2013).

A major factor in the predications of forest-based C sequestration is optimal harvest age, which has also been a primary focus of plantation forest management for several decades (Chang, 1984; Lu and Gong, 2003). The impacts of incentives, subsidies and taxes related to C sequestration and GHG emissions on the optimal harvest age have been widely examined, most notably within a Hartmann modeling framework (Creedy and Wurzbacher, 2001; Hartman, 1976; Stainback and Alavalapati, 2002; Susaeta et al., 2014; van Kooten et al., 1995). Although most of these studies found higher optimal harvest age with positive forest C prices, leading to a higher amount of C sequestered in tree biomass, other studies have indicated that the optimal harvest age may be shorter with C prices. Much of the difference in findings appears to be driven by assumptions about: the balance between two functions of the forests, i.e., sequestering C or postponing sequestered C release (Akao, 2011), the type of C process and selection of discount rates (Chladnà, 2007), the type of C credit payment schemes (Guthrie and Kumaeswaran, 2008), and the incorporation of carbon stocks in dead organic matter (Asante et al., 2011).

Research specifically related to optimal harvest age with CC has typically assumed increasing C prices associated with efficient climate change mitigation policies (Ekholm, 2016; Köthke and Dieter, 2010; Yu et al., 2014). Economic studies that have considered changes in forest growth induced by fluctuating climatic conditions are very limited, primarily because of inherent limitations in growth and yield models used for these analyses, which have traditionally assumed similar or even fixed climatic conditions over time. A notable exception is the work by Ferreir et al. (2016) that used a processed based model coupled with a dynamic programming approach to determine the implications of climate change on eucalyptus stand management in Portugal. Much work remains to be done in this area.

In this study, we focus on the impacts of CC on pine plantations, and use slash pine (Pinus elliottii) in the southern US as an example case. We integrated a process-based model (3-PG, Physiological Processes Predicting Growth, Landsberg and Waring, 1997) with a generalized stand level economic optimization model that accounts for timber and C prices (Susaeta et al., 2014) to determine optimal even-aged forest management for slash pine across different sites in the southern US under a current and two future climatic scenarios. Slash pine is major commercial species in the southern US. It has been planted on 4.2 million ha in the region and extends across a wide range. It is found as far west as eastern Texas, as far north as southern North Carolina, and as far south as south-central Florida (Barnett and Sheffield, 2004). Notably, our approach is sensitive to climatic variables that determine the impacts of future climate conditions on LEVs and optimal harvest ages. As such, we also present an extension of our model – Pressler’s indicator rate formula – to determine the impacts of relative changes of timber production, quality of forest products, and carbon and stumpage prices on the marginal economic returns of slash pine forests. This is particularly relevant in a CC context since changes in forest growth may modify harvesting decisions and significantly affect economic revenues for forest landowners, the supply of wood products, and carbon storage.

Below we specify a generalized carbon sequestration economic model and Pressler’s indicator rate formula, describe the 3-PG process based model, and outline the climatic scenarios used in our analysis. We then describe slash pine forest management, the location of our representative study sites, model parameters, and our application of the generalized economic model. We report the economic impact of future climates on the profitability and current optimal harvest ages of slash pine, and discuss our findings. Finally, a concluding section summarizes the main findings and present avenues for further research.

**Model specification**

**The generalized economic model**

We employed a generalized carbon sequestration economic model to analyze the impact of carbon taxes and subsidies on optimal harvest management (Susaeta et al., 2014; see Appendix B (Eqs. (B1)–(B3) for model derivation)). This model is an important extension of the standard carbon sequestration economic model proposed by van Kooten et al. (1995), which assumes: (i) yearly payments to forest landowners for carbon sequestered by the trees as the forest stand grows and (ii) a tax (cost of carbon release) levied at the time of harvest and due to forest product decay (Appendix A). Unlike the standard approach, the generalized approach assumes that economic factors (stumpage prices, carbon prices, regeneration costs, and discount rates) and biological factors (forest growth, and fraction of carbon that is permanently stored in forest products and landfills) may change from timber crop to timber crop. As such, the generalized model avoids a major complaint of the standard model – its limited ability to account for landowner adaptation and responsiveness to market conditions. For example, with the generalized model, forest landowners may replant a different timber crop for high value forest products, with other wood properties, and a different silviculture intensity – all of which are static in a standard model.

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1 Southern forests also provide other critical ecosystem services (e.g. water availability, biodiversity, wildlife habitat, and recreation) that are essential for society. For example, forests produce 34% of the water yield in the Southern US (Lockaby et al., 2013) and host >1000 native terrestrial vertebrate species (Trani Griepp and Collins, 2013).
Using this model, the land expectation value at the beginning of the first timber crop $LEV_1$ can be represented as follows:

$$LEV_1 = \left\{ -C_t e^{r_1 t_1} + P_t(t_1)Q(t_1) + e^{r_1 t_1} \int_0^{t_1} P_t(t_1) \frac{dQ(t_1)}{dt} e^{-r_1 t} dt_1 - P_c \alpha_1 Q(t_1) [1 - \beta_1] \right\} e^{-r_1 t_1} + e^{r_1 t_1} LEV_2$$

(1)

where $P_t(t_1)$ is the stumpage price of $t_1$ year old trees at the first timber crop; $Q(t_1)$ and $(dQ(t_1))/dt_1$ are, respectively, the merchantable volume and its derivative of a $t_1$ year old forest stand from the first timber crop; $P_c$ is the carbon price and $\alpha_1$ is the amount of carbon per volume of wood (conversion factor); $C_t$, $r_1$, and $\beta_1$, are respectively, regeneration costs, the discount rate and picking factor (proportion of carbon permanently stored in forest products and landfills) for the first timber crop; and $LEV_2$ is the land expectation value at the beginning of the second timber crop (or at the end of the first timber crop). Discounted annual payments for carbon sequestered as the forest grows and carbon taxes are represented, respectively, by $\int_0^{t_1} P_c \alpha_1 dt_1 e^{-r_1 t_1} dt_1$, and $P_c \alpha_1 Q(t_1) [1 - \beta_1] e^{-r_1 t_1}$ for the first timber crop.

Some relevant information is found in Eq. (1). Stumpage prices can fluctuate as the trees age, since a different proportion of forest products is produced. Different timber crops may have different growth rates. Carbon prices are not a function of the forest stand age, but a commodity (i.e., the price of a single ton of carbon is the same regardless if it is sequestered by young or older trees, or by a different timber crop). This model also assumes that regeneration costs, discount rates and the picking factor may fluctuate between timber crops but independently of age of the trees. The land expectation value at the beginning of the first timber crop $LEV_1$ can be equal, greater or lower than the land expectation value at the beginning of the second timber crop $LEV_2$. As such, the harvest age would not necessarily be the same at the beginning and end of the rotation (Susaeta et al., 2014). Finally, each subsequent year’s $LEV_1$ is a function of the previous year’s, such that $LEV_2$ is a function of $LEV_3$, $LEV_3$ is a function of $LEV_4$, and so on. This recursive problem can be solved analytically, i.e. assuming that future land values represent a single factor. For simplicity we present only the key relationship used to determine the optimal harvest age for any $k$ timber crop (see Appendix C or Susaeta et al. (2014)) for the full derivation of the rule of harvest. Thus:

$$\frac{\partial V_k(t_k)}{\partial t_k} + P_c \alpha_k \frac{\partial Q_k(t_k)}{\partial t_k} - \frac{\partial E_k(t_k)}{\partial t_k} = r_k [V_k(t_k) - E_k(t_k)] + r_k LEV_{k+1}$$

(2)

where the stumpage value $V_k(t_k) = P_k(t_k)Q_k(t_k)$, and carbon tax $E_k(t_k) = P_c \alpha_k Q_k(t_k)[1 - \beta_k]$. The left hand side of Eq. (2) represents the net marginal revenues from timber and carbon benefit due to delaying harvest by one additional year. The right hand side of Eq. (2) represents the net marginal cost of delaying the harvest by one additional year due to value of holding the trees (net of carbon tax emissions) plus the value of holding the land. Eq. (2) provides a generalizable rule of harvest: for the $k$th timber crop, if the left hand side (marginal revenues of waiting one year) is greater than the right hand side (marginal cost of waiting one year) of Eq. (2), the harvest should be postponed for one more year. When the right hand side equals the left hand side of Eq. (2), the forest stand should be harvested. From Eq. (2) it is apparent that the harvest decision depends on current benefits due to timber production and carbon sequestration, and future land values. Therefore, future land values can act as a threshold value to determine optimal harvest decisions. Rearranging Eq. (2) and defining threshold values as

$$\phi(t_k) = \left[ \frac{\partial V_k(t_k)}{\partial t_k} + P_c \alpha_k \frac{\partial Q_k(t_k)}{\partial t_k} - \frac{\partial E_k(t_k)}{\partial t_k} - r_k [V_k(t_k) - E_k(t_k)] \right] / r_k,$$

the forest stand would be left to grow one extra year if $LEV_{k+1} < \phi(t_k)$.

**Pressler’s indicator rate formula**

We employed Pressler’s indicator rate formula (Pressler, 1860) to determine the impacts of climate change on the marginal forest stand value and the optimal harvest age. Specifically, we extended Pressler’s indicator rate formula developed for Chang and Deegen (2016) to account for payments for carbon sequestration and carbon tax emissions under a generalized framework. Our approach involves separating the net marginal value of the forest stand into several increments: (i) quantity increment (timber volume growth); (ii) timber price increment (changes in stumpage prices); (iii) quality increment (changes in the proportion of forest products); (iv) carbon quantity increment (total carbon biomass growth); and (v) carbon price increment (changes in carbon prices). For simplicity, we present the key relationships that capture the impacts of climate change on the different variables of our economic model. The full derivation of Pressler’s indicator rate formula can be found in Appendix E.

In Eq. (3) we present the marginal stumpage value over time (MSV) due to timber quantity and quality increment, and timber price increment (Chang and Deegen, 2016). We assume that the forest stand can produce $n$ forest products ($j=1, \ldots, n$), such that $W_{ij}(t_1)$ represents the percentage of the product class $j$ in the timber volume from the first timber crop. Thus:

$$MSV = \sum_{j=1}^{n} \left[ P_{ij}(t_1) W_{ij}(t_1) \frac{\partial Q(t_1)}{\partial t_1} + \sum_{j=1}^{n} P_{ij}(t_1) \frac{\partial W_{ij}(t_1)}{\partial t_1} Q(t_1) + \sum_{j=1}^{n} \frac{\partial P_{ij}(t_1)}{\partial t_1} W_{ij}(t_1) Q(t_1) \right]$$

(3)

where the three terms on the RHS of Eq. (3) represent the timber quantity increment, quality increment, and timber price increment, respectively. Intuitively, $MSV = \frac{\partial V(t)}{\partial t}$.

We also now assume that the carbon price can fluctuate between timber crops, for example due to the introduction of a new environmental policy or changes in carbon markets (i.e., we do not consider that the carbon price is a function of stand age). This is reflected in the price level variable $d$ – a larger $d$ implies a higher carbon price.\(^2\)Increments in carbon prices, carbon sequestered by trees, and proportion of forest products (each forest product is taxed depending on its emission rate or picking factor) also affect the marginal value of the

\(^2\)See Susaeta et al. (2014) for the comparative static analysis of current and future increases in carbon price and their impacts on the optimal harvest age.
carbon tax emission. Thus, the net marginal value of the forest stand due to payments for carbon sequestration and changes in the carbon biomass (MVC) is as follows:

\[
MCV = P_c d_1 \alpha_1 \frac{\partial Q_t(t_1)}{\partial t_1} + P_c \alpha_1 \frac{\partial Q_t(t_1)}{\partial t_1}
\]

\[- \left\{ \sum_{j=1}^{n} \left[ P_c d_1 \alpha_1 W_j(t_1) Q(t_1) \right] \left[ 1 - \beta_{ij} \right] + P_c d_1 \alpha_1 W_j(t_1) \frac{\partial Q_t(t_1)}{\partial t_1} \left[ 1 - \beta_{ij} \right] + P_c d_1 \alpha_1 \frac{\partial W_j(t_1)}{\partial t_1} Q(t_1) \left[ 1 - \beta_{ij} \right] \right\}
\]

(4)

The first two terms on the RHS of Eq. (4) represent, respectively the carbon price increment and the carbon quantity increment, while the last three terms between \{ \} represent the net marginal value of the carbon tax emission increments due to changes in carbon prices, carbon quantity, and proportion of forest products, respectively \( \frac{\partial (Q(t_1))}{\partial t_1} \). Dividing both sides of Eqs. (3) and (4) by the stumpage value \( V_j(t_1) \) and adding them up result in the LHS of Eq. (2), with the caveat we are considering that carbon prices may fluctuate between timber crops. Thus, we have at the optimal harvest age \( t_1 \):

\[
\left( \frac{u}{u+1} \right) = t_1; \quad \text{with } u = \frac{V(t_1) - E_1(t_1)}{LEV_2}
\]

(5)

The LHS of Eq. (5) represents Pressler’s indicator rate formula that can be applied to any timber crop. The first three terms represent the rates of timber quantity, quality, and price increment, respectively, while the fourth and fifth terms represent the rates of carbon quantity, and carbon price increment, respectively. Finally, the terms between \{ \} represent the rates of carbon price, carbon quantity, and quality tax increment. The sum of all rates of increment represent the change of the net marginal value of the forest stand due to timber production and carbon sequestration. Separating the different rates of variable increments in our generalized model will allow us to determine the impact of changes of each of the model variables due to future climatic conditions on the marginal economic revenues for a timber crop. This indicator is also an alternative approach to determine the optimal harvest age. If the indicator rate is greater than the discount rate the forest stand should be left grown one year, otherwise the stand should be harvested.

Forest growth simulations

To estimate slash pine stand growth under different climatic scenarios we used the forest simulation model 3-PG (Physiological Processes Predicting Growth; Landsberg and Waring, 1997). The model was parametrized and successfully validated for slash pine plantations growing across a wide range of ages and stand characteristics in the southern U.S. (Gonzalez-Benecke et al., 2014). Key inputs for the 3-PG model are mean monthly radiation, temperature and precipitation. A full description of the 3-PG model can be found in (Landsberg and Sands, 2011; Landsberg and Waring, 1997).

Additionally, our model included in the computation of merchantable volume partitioning in three forest products: sawtimber (sw), chip-and-saw (cns), and pulpwod (pw). We used the equations reported by Pienaar et al. (1996), and defined three forest products based on quadratic mean diameter (Dq) and merchantable diameter limit: sawtimber (Dq = 29.2 cm; top diameter = 17.8 cm), chip-and-saw (Dq = 19.1 cm; top diameter = 15.2 cm), and pulpwod (Dq = 11.4 cm; top diameter = 7.6 cm). Finally, we also included the mass of top, branches, and foliage (rs) as another carbon pool for the economic analysis.

Climatic scenarios

We ran the model under 3 climate change scenarios: baseline (assuming no changes in climate and CO2 concentration), and two representative concentration pathways (RCPs) used in the IPCC’s Fifth Assessment Report (IPCC, 2013b) to represent greenhouse emission trajectories over time: RCP4.5 (low-to-medium greenhouse gas emissions) and RCP8.5 (high greenhouse gas emissions). For each RCP scenario, we used the second generation Canadian Earth system model (CanESM2) to generate future climate data, downscaled using the Multivariate Adaptive Constructed Analogs (MACA) approach (University of Idaho, 2013). We obtained future projections of temperatures and precipitation for RCP4.5 and RCP8.5 for 60 years (between years 2050 and 2100) and used them as inputs to 3-PG to model slash fine growth for the same period (Table 1). Likewise, we employed the 3-PG model considering historical levels of precipitation and temperatures between years 1950 and 2005 (baseline scenario). Thus, for each of the 3 climatic scenarios we simulated slash pine growth from planting up to age 60 years on 11 sites covering the natural distribution of this species in the southern US (Fig. 1).

Forest management and economic parameters

For each site, the simulations were performed assuming 3 levels of productivity (low, medium and high, defined as site indexes 20, 24 and 28 m, respectively) and three levels of initial planting densities (TD = 750, 1500 and 2250 trees ha\(^{-1}\)). The total number of scenario permutations (climate x SI x TD) was 27. For timber value, we used the average real stumpage prices (2015 base Producer Price Index,
logging industry; USDL Bureau of Labor Statistics, 2015) for sawtimber, chip-and-saw, and pulpwood between 2010 and 2015 (Timber Mart-South, 2011, 2012, 2013, 2014, 2015, 2016) for each of the states in which the sites were located (Table 2).

For the value of carbon, we used the average real auction settlement prices of C between 2010 and 2015 (2015 base, GDP deflator; USDL Bureau of Labor Statistics, 2015) of the California’s cap-and-trade program (California Environmental Protection Agency Air Resources, 2016) (Table 2). The pickling factors for sawtimber, chip-and-saw, and pulpwood were based on Stainback and Alavalapati (2002) and

Table 1
Historical and future average annual temperatures and precipitation for southern sites.

<table>
<thead>
<tr>
<th>Sites</th>
<th>Historical</th>
<th>RCP4.5</th>
<th>RCP8.5</th>
<th>Historical</th>
<th>RCP4.5</th>
<th>RCP8.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max  C</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td></td>
<td>mm yr$^{-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>10.9</td>
<td>27.2</td>
<td>13.2</td>
<td>28.0</td>
<td>13.9</td>
</tr>
<tr>
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<td>28.8</td>
<td>16.9</td>
<td>30.6</td>
<td>18.7</td>
<td>31.5</td>
<td>19.4</td>
</tr>
<tr>
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<td>27.5</td>
<td>14.6</td>
<td>29.5</td>
<td>16.6</td>
<td>30.3</td>
<td>17.3</td>
</tr>
<tr>
<td>Jefferson FL</td>
<td>26.6</td>
<td>13.5</td>
<td>28.6</td>
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<td>16.2</td>
</tr>
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<td>25.7</td>
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<td>27.7</td>
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<td>15.5</td>
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<td>17.2</td>
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<tr>
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<td>12.2</td>
<td>28.2</td>
<td>15.2</td>
<td>30.0</td>
<td>16.9</td>
</tr>
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</tr>
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<td>15.1</td>
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<td>28.1</td>
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<td>28.7</td>
<td>16.3</td>
</tr>
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</table>

Fig. 1. Location of the slash pine sites in the southern US.

Table 2
Average prices of carbon and state stumpage prices (2010–2015), pickling factors and total regeneration costs for slash pine forests.

<table>
<thead>
<tr>
<th>Sites</th>
<th>State</th>
<th>Stumpage price $ m$^{-3}$</th>
<th>$P_{ww}$</th>
<th>$P_{ww}$</th>
<th>$P_{ww}$</th>
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<td>19.7</td>
<td>12.4</td>
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</tr>
<tr>
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<td>LA</td>
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<td>19.3</td>
<td>11.5</td>
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<td>SC</td>
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<td>20.2</td>
<td>10.3</td>
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<tr>
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<td>TX</td>
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<td>17.2</td>
<td>10.9</td>
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<table>
<thead>
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<th>Price of carbon</th>
<th>$Mg^{-1}$</th>
<th>Pickling factor</th>
<th>$\beta_{ww}$</th>
<th>$\beta_{ww}$</th>
<th>$\beta_{ww}$</th>
<th>Total cost (SP + W + P + S)</th>
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<td>45</td>
<td>0.8</td>
<td>0.35</td>
<td>0.15</td>
<td>0</td>
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</table>

<table>
<thead>
<tr>
<th>Activity</th>
<th>Cost $/ha^{-1}$</th>
<th>TD trees $ha^{-1}$</th>
<th>Seedling cost ($)</th>
<th>Total cost (SP + W + P + S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regeneration costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Site preparation (SP)</td>
<td>237</td>
<td>750</td>
<td>37</td>
<td>475</td>
</tr>
<tr>
<td>Weed control (W)</td>
<td>91</td>
<td>1500</td>
<td>75</td>
<td>512</td>
</tr>
<tr>
<td>Planting (P)</td>
<td>109</td>
<td>2250</td>
<td>112</td>
<td>550</td>
</tr>
<tr>
<td>Seeding ($seedling^{-1}$)</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Suseta et al. (2014). The amount of carbon per volume of wood ($\alpha$) was set to 0.27 (Suseta et al., 2014). We also assumed that the harvesting residues (top, branches and foliage) are burned immediately after harvesting – all carbon is released to the atmosphere, thus, the pickling factor $\beta_0$ is set to 0. Regeneration costs were based on site preparation, weed control at the moment of planting, planting costs, and seedling cost based on initial planting density (Table 2; Barlow and Levendis, 2015). The real discount rate was assumed to be 4%.

The generalized carbon sequestration model is able to explore the implications of future land values on the current harvesting decision. Using the model, we first applied the traditional carbon sequestration model (Eq. (A1)) to obtain the LEVs and optimal harvest ages for all sites, climatic scenarios, and combinations of SI and TD. The land value for the baseline scenario is captured by LEV$_1$ while the land values for the future climatic scenarios RCP4.5 and RCP8.5 are reflected in LEV$_2$ in Eq. (1). Next, we employed Eq. (2) to determine the impacts of futures LEVs given by changes in climate – those for RCP4.5 and RCP8.5 – on the optimal forest management for the baseline scenario. Finally, we employed Eqs. (3)–(5) to determine the rate of increments of each variable of the economic model, the change in the net marginal economic revenues due to timber production, and carbon sequestration under changing climatic conditions.

**Results**

**LEVs and climatic scenarios**

The net average impact of climate change on the 11 sites was negligible, but with a tremendous variation across the sites. Averaging across all sites and combinations of SI and TD, we found that the LEV$_1$ was $6314.7$ ha$^{-1}$ while the LEV$_2$s for RCP4.5 and RCP8.5 were $6328.6$ ha$^{-1}$ and $6318.4$ ha$^{-1}$, respectively (Fig. 2). However, the LEVs for all climatic scenarios (LEV$_1$s and LEV$_2$s) showed a wide variation in magnitude, ranging approximately between $1124.5$ and $14,446.4$ ha$^{-1}$ (Fig. 2). Likewise, compared to the baseline climatic scenario, the average harvest ages for all sites remained roughly the same (~31 years) for both moderate (RCP4.5: 32 years) and extreme climatic scenarios (RCP8.5: 31 years). However, optimal harvest ages fluctuated between 20 and 43 years for all climatic scenarios (Fig. 2; Table 3).

In just five sites, moderate changes in climate (scenario RCP4.5) resulted in higher average LEV$_2$s. These included Brantley ($5707.7$ ha$^{-1}$), Colleton ($5503.5$ ha$^{-1}$), Covington (AL) ($5792.2$ ha$^{-1}$), Covington (MS) ($6169.9$ ha$^{-1}$), and Santa Rosa ($7928.8$) – representing a 5%, 7%, 6%, 4%, and 3% increase compared to the their respective LEV$_1$s (Fig. 3). Five sites generated higher LEV$_2$s with extreme changes in climate (scenario RCP8.5): Colleton ($5539.9$ ha$^{-1}$), Covington (AL) ($6080.3$ ha$^{-1}$), Covington (MS) ($6214.9$ ha$^{-1}$), Santa Rosa ($7977.5$ ha$^{-1}$), and
Table 3
The current and future average land expectation values (LEV₁, LEV₂) and land expectation values due to carbon payments (LEVc₁, LEVc₂) under different climatic scenarios, SI and TD.

<table>
<thead>
<tr>
<th>SI m</th>
<th>TD trees ha⁻¹</th>
<th>Baseline LEV₁ $ha⁻¹</th>
<th>RCP4.5 LEV₁ LEV₂ $ha⁻¹</th>
<th>RCP8.5 LEV₁ LEV₂ $ha⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>750</td>
<td>2151.3</td>
<td>2842.6</td>
<td>1677.1</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>1917.8</td>
<td>2335.2</td>
<td>1508.6</td>
</tr>
<tr>
<td></td>
<td>2250</td>
<td>1796.4</td>
<td>2148.7</td>
<td>1569.4</td>
</tr>
<tr>
<td>24</td>
<td>750</td>
<td>7758.6</td>
<td>7410.2</td>
<td>3205.2</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>6490.8</td>
<td>6264.3</td>
<td>3215.1</td>
</tr>
<tr>
<td></td>
<td>2250</td>
<td>5877.3</td>
<td>5710.8</td>
<td>3346.7</td>
</tr>
<tr>
<td>28</td>
<td>750</td>
<td>11,658.6</td>
<td>11,630.1</td>
<td>4510.0</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>9980.3</td>
<td>9746.5</td>
<td>4290.3</td>
</tr>
<tr>
<td></td>
<td>2250</td>
<td>9201.3</td>
<td>8868.7</td>
<td>4253.9</td>
</tr>
</tbody>
</table>

* A full report of LEV and LEVc per site is available from authors upon request.

Fig. 3. The current and future average land expectation values (LEV₁, LEV₂) and optimal harvest ages (T, on top of each bar) under different climatic scenarios. A complete report for each combination of TD and SI is available from authors upon request.

Washington ($6826.9 ha⁻¹) – reflecting a 6%, 11%, 5%, 3% and 2% increase compared to the LEV₁. Notably, four of these sites experienced higher LEVs under both climate change scenarios. The best site’s economic performances under climate change were Santa Rosa, and Polk for scenario RCP4.5, and Santa Rosa and Jefferson for scenario RCP8.5 (Fig. 3). On the contrary, sites such as Dolly and Colleton, and Dolly and Brantley yielded the lowest LEV₂ for both climatic scenarios.

Notably, 48% of the total LEVs for all climatic scenarios are due to payments for carbon stored in the aboveground portion of the trees (Table 3). The profitability of slash pine only increased with climate change at low levels of SI. On average for SI = 20 m, all sites, and TD = 750, 1500 and 2250 trees ha⁻¹, the LEV₂'s for RCP4.5 were 32%, 22% and 20% greater than the LEV₁ (Table 3). With scenario RCP8.5, the difference in economic revenues was accentuated: the LEV₂'s were 32%, 29% and 27% higher. Although future land values increased when slash pine was planted in low productivity conditions, the contribution of carbon payments to the total LEV decreased, on average, from 68% (baseline) to 65% and 63% for both climatic scenarios, all sites and TDs. Greater changes in LEVs were obtained for those sites located in northeastern distribution range (Fig. 4), reflecting the differential impacts of temperature changes. On average, a 1 °C increase generated increases in land values of $350.2 ha⁻¹ and $334.4 ha⁻¹ for Covington (AL) and Covington (MS), respectively, for scenario RCP4.5 and TD = 1500 trees ha⁻¹.

On the contrary, lower future land values were obtained when planting slash pine in medium and high productivity conditions. For SI = 24 m, and TD = 750, 1500 and 2250 trees ha⁻¹, the average LEV₂'s decreased by the same rate – 4%, 3% and 3% – for scenarios RCP4.5 and RCP8.5 compared to the average LEV₁ (Table 3). For SI = 28, and TD = 750, 1500 and 2250 trees ha⁻¹, the profitability of slash pine remained the same, and decreased by 2%, and 4%, respectively, for scenario RCP4.5 while it decreased at higher rate for scenario RCP8.5: 2%, 4% and 4%. The proportion of carbon benefits to the total LEV for medium and high SI remained at the same levels as the current climatic conditions for scenarios RCP4.5 (50%) and RCP8.5 for all TDs (44%). The southeastern sites showed the greatest increases in LEV with changes in temperature (Fig. 3). For example, on average, in the case of Polk, per 1 °C increase the LEV decreased by $460.2 ha⁻¹ and $116.4 ha⁻¹ for SI = 24 m and SI = 28 m, respectively. With moderate forest productivity conditions (SI = 24 m), the northeastern sites such as Covington (AL, MS), and Colleton showed increases in economic rents with changes in temperatures (Fig. 4).
Optimal current harvest ages and future climatic scenarios

One of the key features of our model is that it helps determine the impact of future climates on the current harvesting decision. According to our mathematical model, with increased (decreased) future values, the right-hand-side of Eq. (2) would increase (decrease) thus the left-hand-side would have to increase (decrease) to restore the equality, shortening (lengthening) the current harvest age. The mathematical proofs for the comparative static analysis on an increase in the future forest growth can be found in Appendix D. See Susaeta et al. (2014) for the comparative statics of the other parameters of the generalized carbon sequestration model.

Table 4 presents the optimal harvest ages and thresholds for slash pine for all sites, TD = 1500 and SI = 24 m. For example, in the case of Polk, the LEV1 was $7695.5 ha⁻¹ and the current optimal harvest age was 31 years. At that site, the LEV2 for RCP4.5 was $6857.9 ha⁻¹. If future LEV2 were $6857.9 ha⁻¹ while the current land value remained at $7695.5 ha⁻¹, the optimal current harvest age would have to be lengthened 2 years (from 31 to 33 years) since the LEV2 is between the threshold values $φ_1$ 5595.4 and 6898.6 ha⁻¹. For Colleton, the LEV1 was $5282.9 ha⁻¹ and the current optimal harvest age was 31 years. Under Scenario RCP4.5, the LEV2 of $5491.4 ha⁻¹ reduced the current harvest age from 31 to 30 years, since that site’s LEV2 is between the threshold values $φ_1$ $5452.1–5561.5$ ha⁻¹. We found contrary results for Brantley and Covington (AL) for Scenario RCP4.5 and Washington for scenario RCP8.5 – the current harvest age increased with increased LEV2s – probably due to the violation of a concavity assumption for the forest growth function, which we address in the Discussion section.

Rates of quantity, quality and price increments and marginal returns under climate change

Table 5 shows the changes in volume growth, stumpage prices, and prices of carbon on the land expectation value under moderate climatic conditions (RCP4.5) for our example site of Covington (MS). We selected this site for our analysis since it represents the northern distribution of slash pine where the greater changes in LEV2s are obtained under climate change. Here, moderate changes in climatic conditions have a little impact on the changes of timber quantity, type of forest products, and carbon quantity. At age 35 years, if timber production increased to 271.41 m³ the next time period (Table 5a) the net rate of marginal revenues for the slash pine forest stand would increase by 1.64% (all else equal) (Table 5b). Similarly, the net rate of marginal economic returns would increase 0.92% if the proportion of sawtimber, chip and saw, and pulpwood would increase to 0.26%, 47.64%, 52.10% the following year, respectively (Table 5b). Between ages 33 and 36 years, the rate of quantity increment of timber production and production of forest products slightly decreases on an average
rate of 1.76% and 0.95%, respectively. In the case of carbon production, the rate of quantity increment shows a fluctuating trend averaging a 2.35% increase in the net rate of marginal economic returns for the forest stand for the same time period.

Changes in timber markets have a stronger impact on the economic returns than changes in timber quality and quantity, yet the direction of the effect is not definitive. At age 33 years, given the increase in the price of sawtimber, chip and saw, and pulpwood to $25.98 m⁻³, $16.88 m⁻³, and $9.74 m⁻³ for the next period (Table 5a), respectively, we see a positive rate of stumpage price increase (5.34%) (Table 5b). However, at age 34 years, a fall in the prices of sawtimber ($25.88 m⁻³ to 25.45 m⁻³), chip and saw ($16.88 m⁻³ to $16.08 m⁻³) and pulpwood ($9.74 m⁻³ to $8.63 m⁻³) for the next period causes a negative rate of price increment of -7.56% (Table 5b). A negative rate of price increment is also obtained at age 35 years (-5.04%). The decrease in the stumpage prices for chip and saw and pulpwood – from $16.09 m⁻³ to $15.11 m⁻³, and from $8.64 m⁻³ to $8.37 m⁻³ between ages 36 and 35 years – together with the higher proportion of these two forest products work to offset the increase in the price of sawtimber (from $25.45 m⁻³ to $27.14 m⁻³). Between ages 33 and 36 years, the changes in the rate of growth in stumpage prices would cause a drop of the net rate of marginal economic revenues by 0.20%.

Carbon markets have less impact on the economic returns compared to the effects of climate change-related variations in the quantity of carbon sequestered by slash pine forests. With the exception of age 33 years when the carbon price decreases (from $49.02 Mg⁻¹ to $44.19 Mg⁻¹, Table 5a), the rate of carbon price increment remains positive, averaging a low 0.08%. The mixed impact of climate change (quantity of carbon produced and quality of the forest products at time of harvesting) and carbon markets (changes in carbon taxes) have a strong impact on the total changes in the rate of carbon tax increments (Table 5c). On average, changes in carbon tax increments decrease the net rate marginal economic returns by 5.36% for the time period 33 and 36 years.

On average, if the slash pine forest were harvested between ages 33 and 36 years, the total net rate of marginal economic returns, including the sum of all rates of increments of prices, quality and quantities and carbon taxes, would increase by 2.83% with moderate changes in climatic conditions. This approach also indicates the timing of optimal harvest age, which is reached when the indicator rate is equal to or less than the discount rate. Not surprisingly, the indicator rate is –10.16% at age 34 years when the LEV2 reaches its maximum value ($2168.89 ha⁻¹) (Table 5c).

Discussion

Our findings suggest that land average values are not strongly affected by predicted changes in climatic conditions, with notable exceptions for slash pine planted in low productivity sites. The negligible impacts on economic rents were reflected by minimal changes in the overall biomass production and proportion of forest products. For example, for SI = 24, and scenarios RCP4.5 and RCP8.5, and on average for all sites, the production of sawtimber decreased by 3% and 4%, respectively, compared to the production of timber under current climatic conditions after 35 years. Likewise, the production of residues remained the same for both future climatic scenarios RCP4.5 and RCP8.5. We caution that our findings are specific to the southern U.S., and other areas may experience very different results. For example, climate change is expected to decrease the land values of European forestlands by 50% by 2100 due to the loss of valuable forest species (Hanewinkel et al., 2013).
In low productivity sites (SI = 20), we found that changing climatic conditions improved the productivity of slash pine and associated economic rents. With higher productivity, we predict 15% and 18% higher average overall biomass production, 62% and 79% higher chip- and saw production, and 165-fold and 607-fold increase in sawtimber for scenarios RCP4.5 and RCP8.5, respectively, across all sites. Despite the large percentage increase in sawtimber production, absolute increases were not large. For example, in Covington (MS), the production of sawtimber increased from 0.01 m$^3$ ha$^{-1}$ to 0.53 m$^3$ ha$^{-1}$ (RCP4.5) and 1.5 m$^3$ ha$^{-1}$ (RCP8.5) at age 35 years.

Regardless of site productivity, the land values under current climatic conditions were in general greater in the southeastern distribution of slash pine, where slash pine has been favored due to historical warmer temperatures and increased precipitation levels decreasing the risk of water stress. With future assumed climatic conditions, we found higher land values at sites in the southern versus the northern distribution of slash pine, but the rate of change in these values was more pronounced in the northern sites. This suggests that the economically viable range of slash is increasing northward with climate change.

We also found, as expected, that higher (lower) future land values would shorten (lengthen) the current harvest age for slash pine in most of the sites. Higher productivity leads to increases in land values, which incentivizes forest landowners to harvest earlier and invest in forest regeneration to obtain greater economic returns in the near future. If landowners experienced a net decrease instead of a net increase in productivity, then they could compensate by delaying the current harvest age. We produced contrary results for two sites, probably due to the violation of concavity assumption for the forest growth function. Although it is widely assumed that the growth function is concave at stand age, our Faustmann model based approach could in theory have multiple optima and therefore the rotation age may fall in the interval when the forest growth is convex, i.e., an interior local minimum solution (Gong and Lofgren, 2016). For example, in the case of Brantley, the LEV$_2$ was $5826.4$ ha$^{-1}$ (RCP4.5) with an optimal harvest age was 36 years (Table 3), at which the second derivative of the growth function is positive (Fig. 5).

In our analysis, stumpage prices slightly decrease due to market conditions without the effects of climate change. We also find that the rate of the carbon price increment through the years is negligible. With changing climatic conditions, stumpage prices are projected to increase at an annual rate of 0.4% for the following 100 years in the U.S., representing a 15% fall relative to a no climate change baseline (Sohngen and Tian, 2016). Other projections suggest that the nation’s timber prices would increase by 30% by 2030 (Buongiorno et al., 2011). Ongoing development of carbon markets in the U.S. (for example, the California AB 32 cap-and-trade program), and stronger carbon prices (as a function of the global warming potential as suggested by Price and Willis (2016)) are expected. As such, our model suggests that the investment in near-term forest would increase in light of higher future forestland values in the region.

The application of our generalized model provides richer insights about policy implications under a context of climate change. If the goal is to invest in current private reforestation and strengthen carbon markets, only policies that have a long-term effect on carbon prices or forest sinks are preferred. Strong future carbon prices encourage new actors to enter these markets, invest in reforestation, and play a key role in forest conservation in light of a future positive economic environment. This is particularly relevant since securing economic sustainability of forestlands is essential to avoid future losses of forestlands, a phenomenon likely to occur in the region, mainly due to urbanization (Wear and Greis, 2012), changes in the dynamics of forest ownership such as fragmentation of family forest lands and lack of interest in maintain forest legacy (Butler and Wear, 2013), and disturbance risk (Suseta et al., 2016a, 2016b).

**Concluding remarks**

This paper explored the implications of two climatic scenarios RCP4.5 and RCP8.5 on the land expectation values of slash pine forest in different sites across the southern US. We employed the 3-PG model for slash pine to include the impacts of climatic variables on the forest growth, and used a generalized model that accounts for timber and carbon prices to determine the effects of changing climates on the current optimal harvest age. Our findings suggested that, on average, for all sites, productivity conditions and tree planting densities, future climate had a minimal impact on LEVs, and optimal harvest ages compared to those for the current climatic scenario. However, we found a wide variation in impacts across all sites. Furthermore, the payments for carbon sequestration contributed to almost half value of the economic revenues.

Our findings also indicated that futures LEVs due to climatic conditions only increased when slash pine was planted in sites with low productivity conditions. This increase in the economic revenues was accentuated with extreme changing climatic conditions (RCP8.5). The sites located in the northeastern distribution range had the greatest increase in LEV$_5$ when the temperature increased by 1 °C with low and moderate forest productivity conditions. Reductions in land values were found when slash pine was planted in medium or high productivity conditions. Furthermore, southeastern sites showed the greatest decreases in LEV when the temperature increased by 1 °C.

![Fig. 5. The second derivative of the timber volume Q’(t) of slash pine in Brantley.](image-url)
Our empirical findings support the assumption that, in general, higher (lower) future land values would shorten (lengthen) the current harvest age for slash pine. Finally, we found that changes in the rates of stumpage and carbon price increment had a stronger impact than changes in timber quality and quantity, and carbon quantity, on the marginal economic revenues for slash pine.

There are several avenues to enhance our research. For example, the inclusion of the impacts on climate change-related disturbances such as wildfires pests, and storms are likely to reduce the forestry economic revenues. Another extension of our study might be the use of a life cycle assessment to evaluate carbon emissions from the silvicultural activities. Finally, the implications of how timber and carbon prices will evolve under changing environmental conditions is also a subject of further analysis.

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Appendix A. The traditional carbon sequestration model

Assume yearly payments to forest landowners for carbon sequestered by the trees as the forest stand grows, and a tax (cost of carbon release) levied at the moment of harvesting and due to forest product decay, the land expectation value LEV takes the following form:

$$LEV(t) = \left[ \int_0^t P_c \alpha \frac{dQ(t)}{ds} e^{-r_s} ds - P_c \alpha Q(t)(1 - \beta) e^{-r_t} \right] / (1 - e^{-r_t}) \quad (A1)$$

where $P$ is the stumpage price, $Q(t)$ and $\frac{dQ(t)}{ds}$ are the merchantable and derivative of the merchantable timber volume at time $t$, $P_c$ is the carbon price, $\alpha$ is the conversion factor, $r_s$ and $\beta$ are, respectively, the regeneration costs, real discount rate, pickling factor (proportion of carbon permanently stored in forest products and landfills).

Appendix B. Derivation of the generalized carbon sequestration model

Assume that $P_i(t_i)$ is the price of stumpage of $t_i$ year old trees at the $i$th timber crop, $Q_i(t_i)$ and $\frac{dQ_i(t_i)}{ds_i}$ are the merchantable volume and derivative of the merchantable function, respectively, of a $t_i$ year old stand from the $i$th timber crop, $C_i$ and $t_i$ are the regeneration cost and discount rate for the $i$th timber crop. Finally, $\alpha_i$ and $\beta_i$ are, respectively, the amount of Mg of carbon per m$^3$ of wood (conversion factor), and pickling factor for the $i$th timber crop. The land expectation value at the beginning of the first timber crop of the for timber crop $LEV_1$ is:

$$LEV_1 = \left\{ -C_1 e^{r_1 t_1} + P_1(t_1)Q_1(t_1) + e^{r_1 t_1} \int_0^{t_1} P_c \alpha_1 \frac{dQ_1(s_1)}{ds_1} e^{-r_1 s_1} ds_1 - P_c \alpha_1 Q_1(t_1)(1 - \beta_1) \right\} e^{-r_1 t_1} + \left\{ -C_2 e^{r_2 t_2} + P_2(t_2)Q_2(t_2) + e^{r_2 t_2} \int_0^{t_2} P_c \alpha_2 \frac{dQ_2(s_2)}{ds_2} e^{-r_2 s_2} ds_2 - P_c \alpha_2 Q_2(t_2)(1 - \beta_2) \right\} e^{-r_2 t_2} - \left( \ldots \right) \quad (B1)$$

$LEV_i$ then denotes the land expectation value at the beginning of the $i$th timber crop; defining the carbon tax value $E_i(t_i) = P_c \alpha_i Q_i(t_i)(1 - \beta_i)$, and rearranging Eq. (B1), we have:

$$LEV_1 = \left\{ -C_1 e^{r_1 t_1} + P_1(t_1)Q_1(t_1) + e^{r_1 t_1} \int_0^{t_1} P_c \alpha_1 \frac{dQ_1(s_1)}{ds_1} e^{-r_1 s_1} ds_1 - E_1(t_1) \right\} e^{-r_1 t_1} + e^{-r_1 t_1}$$

$$\left\{ \sum_{i=2}^{\infty} \left\{ -C_i e^{r_i t_i} + P_i(t_i)Q_i(t_i) + e^{r_i t_i} \int_0^{t_i} P_c \alpha_i \frac{dQ_i(s_i)}{ds_i} e^{-r_i s_i} ds_i - E_i(t_i) \right\} \right\} e^{-r_i t_i} \quad (B2)$$

The expression between |) of the second component on the right hand side (RHS) of Eq. (B2), represents the land expectation value at the beginning of the second timber crop, which we define as $LEV_2$. Thus Eq. (B2) can be rewritten as:

$$LEV_1 = \left\{ -C_1 e^{r_1 t_1} + P_1(t_1)Q_1(t_1) + e^{r_1 t_1} \int_0^{t_1} P_c \alpha_1 \frac{dQ_1(s_1)}{ds_1} e^{-r_1 s_1} ds_1 - E_1(t_1) \right\} e^{-r_1 t_1} + e^{-r_1 t_1} LEV_2 \quad (B3)$$

Appendix C. The rule of harvest of the generalized carbon sequestration model

To determine the optimal conditions for the optimal harvest age, Eq. (B3) can be expressed as:
\[
\begin{align*}
LEV_1 = & \sum_{i=1}^{k-1} \left\{ -C_i e^{\eta i} + P_i(t_i)Q_i(t_i) + e^{\eta i} \int_0^{t_i} P_i \alpha_k \frac{dQ_i(s_i)}{ds_i} e^{-r s_i} ds_i - E_i(t_i) \right\} e^{-\sum_{j=1}^{i} \eta j} \\
+ & \left\{ -C_k e^{\eta k} + P_k(t_k)Q_k(t_k) + e^{\eta k} \int_0^{t_k} P_k \alpha_k \frac{dQ_k(s_k)}{ds_k} e^{-r s_k} ds_k - E_k(t_k) \right\} e^{-\sum_{j=1}^{k} \eta j} + (LEV_{k+1})e^{-\sum_{j=1}^{i} \eta j}
\end{align*}
\]  

\text{(C1)}

Differentiating Eq. (C.1) with respect to \( t_k \) for any \( k \)th timber crop and setting it equal to zero to obtain the maximum \( LEV_1 \) and the optimal harvest age, we have:

\[
\frac{\partial LEV_1}{\partial t_k} = \left\{ -r_k C_k e^{\eta k} + P_k(t_k) \frac{\partial Q_k(t_k)}{\partial t_k} + P_k(t_k) \frac{\partial Q_k(t_k)}{\partial t_k} + r_k e^{\eta k} \int_0^{t_k} P_k \alpha_k \frac{dQ_k(s_k)}{ds_k} e^{-r s_k} ds_k + e^{\eta k} P_k \alpha_k \frac{dQ_k(t_k)}{dt_k} e^{-r t_k} - \frac{\partial E_k(t_k)}{\partial t_k} \right\} e^{-\sum_{j=1}^{k} \eta j} - \left\{ -C_k e^{\eta k} + P_k(t_k)Q_k(t_k) + e^{\eta k} \int_0^{t_k} P_k \alpha_k \frac{dQ_k(s_k)}{ds_k} e^{-r s_k} ds_k - E_k(t_k) \right\} r_k e^{-\sum_{j=1}^{k} \eta j} = 0
\]

\text{(C2)}

Let \( V_k \) be the stumpage value \( V_k(t_k) = [P_k(t_k)Q_k(t_k)] \) and \( \frac{\partial V_k(t_k)}{\partial t_k} = \frac{\partial P_k(t_k)}{\partial t_k} Q_k(t_k) + P_k(t_k) \frac{\partial Q_k(t_k)}{\partial t_k} \). Thus Eq. (C2) can be rewritten as:

\[
\frac{\partial V_k(t_k)}{\partial t_k} + P_k \alpha_k \frac{\partial Q_k(t_k)}{\partial t_k} - \frac{\partial E_k(t_k)}{\partial t_k} = r_k [V_k(t_k) - E_k(t_k)] + r_k LEV_{k+1}
\]

\text{(C3)}

Appendix D. Comparative statics for a higher future forest growth level

Eq. (2) can be re-expressed as

\[
H = \frac{\partial V_k(t_k)}{\partial t_k} + P_k \alpha_k - \frac{\partial Q_k(t_k)}{\partial t_k} - \frac{\partial E_k(t_k)}{\partial t_k} - r_k [V_k(t_k) - E_k(t_k)] + r_k LEV_{k+1}
\]

\text{(D1)}

We derive the total derivative of Eq. (D1) with respect to each future parameters and use the implicit function theorem to determine the impact of a higher future forest growth level. To do this, it is necessary to introduce a forest growth level variable \( z_n \). The effect of a higher future level forest growth level on the current harvest age of the forest stand is reflected by a larger \( z_n \). The implicit function theorem suggests the following relationship for a higher future forest growth level \( z_n \):

\[
\frac{dt_k}{dz_n} = -\left( \frac{\partial H}{\partial z_n} \frac{\partial H}{\partial t_k} \right), \text{ for } n > k
\]

\text{(D2)}

Furthermore:

\[
\frac{\partial H}{\partial t_k} = \frac{\partial^2 V_k(t_k)}{(\partial t_k)^2} + P_k \alpha_k \frac{\partial^2 Q_k(t_k)}{(\partial t_k)^2} - r_k \left[ \frac{\partial V_k(t_k)}{\partial t_k} - \frac{\partial E_k(t_k)}{\partial t_k} - \frac{\partial^2 E_k(t_k)}{(\partial t_k)^2} \right]
\]

\text{(D3)}

We assume that the stumpage value \( V_k(t_k) \) is an increasing and concave function with respect to \( t_k \) thus \( \frac{\partial V_k(t_k)}{\partial t_k} > 0, \frac{\partial^2 V_k(t_k)}{(\partial t_k)^2} < 0 \). \( E_k(t_k) \) also depends on the increasing volume function until it reaches its biological maturity, thus \( \frac{\partial E_k(t_k)}{\partial t_k} > 0 \), and due to the concavity assumption for the volume function \( \frac{\partial^2 E_k(t_k)}{(\partial t_k)^2} < 0 \). Furthermore, \( V_k(t_k) \) is greater than \( E_k(t_k) \) since the stumpage price is generally higher than the price of carbon. Formally \( V_k(t_k) > E_k(t_k) \) if \( P_k(t_k) > P_k \alpha_k [1 - B_k] \), thus \( \frac{\partial V_k(t_k)}{\partial t_k} > \frac{\partial E_k(t_k)}{\partial t_k} \). Finally, with \( \frac{\partial^2 V_k(t_k)}{(\partial t_k)^2} > \frac{\partial^2 E_k(t_k)}{(\partial t_k)^2} < 0 \), \( P_k(t_k) > P_k \alpha_k [1 - B_k] \), the rate of change of the marginal value of the carbon emissions is greater than the rate of change of the marginal stumpage value, i.e., \( \frac{\partial^2 E_k(t_k)}{(\partial t_k)^2} > \frac{\partial^2 V_k(t_k)}{(\partial t_k)^2} \). These relationships ensure that the second derivative of \( LEV_1 \left( \frac{\partial^2 H}{\partial t_k^2} \right) < 0 \). This relationship must hold to represent the second order sufficiency conditions for the maximization of \( LEV_1 \).

Thus, we need to identify the impact of a higher future forest growth level \( z_n \) of the \( n \)th timber crop on the harvest age of the current stand focusing only on future land values \( LEV_{k+1} \). This implies the identification of the sign of \( \frac{\partial H}{\partial z_n} \) to determine if \( \frac{\partial H}{\partial z_n} > 0 \) (lengthen the harvest age) or \( \frac{\partial H}{\partial z_n} < 0 \) (shorten the harvest age). Recall that \( LEV_{k+1} = \sum_{j=k+1}^{\infty} \left\{ -C_i e^{\eta i} + P_i(t_i)Q_i(t_i) + e^{\eta i} \int_0^{t_i} P_i \alpha_i \frac{dQ_i(s_i)}{ds_i} e^{-r s_i} ds_i - E_i(t_i) \right\} e^{-\sum_{j=1}^{i} \eta j} \), where the stumpage value, the cumulative payments for carbon sequestered (integral) and the carbon taxes depend on the forest growth. In order to differentiate the integral \( \int_0^{t_i} P_i \alpha_i \frac{dQ_i(s_i)}{ds_i} e^{-r s_i} ds_i \) with \( i = k + 1 \) of future land values \( LEV_{k+1} \), we use the technique of differentiation.
under the integral sign, i.e., $\frac{d}{dx} \int f(x,w) = \int \frac{d}{dx} f(x,w)$. Introducing a higher future forest growth level variable $z_n$, we have:

$$H = \frac{\partial V_k(t_k)}{\partial t_k} + P_c \alpha_k \frac{\partial Q_k(t_k)}{\partial t_k} - \frac{\partial E_k(t_k)}{\partial t_k} - r_k [V_k(t_k) - E_k(t_k)] - r_k$$

$$\left\{ \begin{array}{l}
C_k e^{\alpha_k t_k} + V_k(t_k) + e^{\alpha_k t_k} \int_0^{t_k} P_c \alpha_k \frac{dQ_k(s)}{ds} e^{-r_k s} ds_k - E_k(t_k) \\
p_n e^{\alpha_k t_n} + z_n V_n(t_n) + e^{\alpha_k t_n} \int_0^{t_n} P_c \alpha_n \frac{dQ_n(s)}{ds} e^{-r_k s} ds_n - z_n E_n(t_n)
\end{array} \right\} e^{-r_k t_k} + \ldots$$

Thus,

$$\frac{\partial H}{\partial z_n} = (-r_k) \left\{ V_n(t_n) + e^{\alpha_k t_n} \int_0^{t_n} P_c \alpha_n \frac{dQ_n(s)}{ds} e^{-r_k s} ds_n - E_n(t_n) \right\} e^{-r_k t_n} < 0, \quad \frac{dt_k}{dz_n} < 0$$

Therefore, a future increase in forest growth would result in a younger harvest age for the current forest stand.

**Appendix E. Derivation of Pressler’s indicator rate formula for the generalized carbon sequestration model**

Following Chang and Deegen (2016), let $W_{ij}(t_1)$ denote the percentage of the forest product $j$ in the stand volume $Q_i(t_1)$, and let $P_{ij}(t_1)$ be the stumpage price of product class $j$. The stumpage value at the time $t_1$ can be expressed as follows:

$$V_i(t_1) = \sum_{j=1}^{n} P_{ij}(t_1) W_{ij}(t_1) Q_i(t_1)$$

(E1)

Thus, the marginal value of the stumpage value (MSV) is:

$$MSV = \sum_{j=1}^{n} \left[ P_{ij}(t_1) W_{ij}(t_1) \frac{\partial Q_i(t_1)}{\partial t_1} + \sum_{j=1}^{n} P_{ij}(t_1) \frac{\partial W_{ij}(t_1)}{\partial t_1} Q_i(t_1) + \sum_{j=1}^{n} P_{ij}(t_1) W_{ij}(t_1) Q_i(t_1) \right]$$

(E2)

Let $P_c \alpha_1 \frac{\partial Q_1(t_1)}{\partial t_1} - \sum_{j=1}^{n} P_c \alpha_1 Q_i(t_1) W_{ij}(t_1) [1 - \beta_{ij}]$ denote the net economic revenues due to carbon sequestration at time $t_1$. Assuming that the carbon price can be increased by a price level variable $d_1$, the marginal value of carbon sequestration (MCV) is:

$$MCV = P_c d_1 \alpha_1 \frac{\partial Q_1(t_1)}{\partial t_1} + P_c \alpha_1 \frac{\partial Q_1(t_1)}{\partial t_1}$$

$$- \left\{ \sum_{j=1}^{n} \left[ P_c d_1 \alpha_1 W_{ij}(t_1) Q_i(t_1) [1 - \beta_{ij}] + P_c d_1 \alpha_1 W_{ij}(t_1) \frac{\partial Q_i(t_1)}{\partial t_1} [1 - \beta_{ij}] + P_c d_1 \alpha_1 \frac{\partial W_{ij}(t_1)}{\partial t_1} Q_i(t_1) [1 - \beta_{ij}] \right] \right\}$$

(E3)

Eq. (2), for the first timber crop, and considering a carbon price level variable $d_1$, can be also expressed as the sum of Eqs. (E2) and (E3). Thus:

$$\sum_{j=1}^{n} \left[ P_{ij}(t_1) W_{ij}(t_1) \frac{\partial Q_i(t_1)}{\partial t_1} + \sum_{j=1}^{n} P_{ij}(t_1) \frac{\partial W_{ij}(t_1)}{\partial t_1} Q_i(t_1) + \sum_{j=1}^{n} P_{ij}(t_1) W_{ij}(t_1) Q_i(t_1) \right] + P_c d_1 \alpha_1 \frac{\partial Q_1(t_1)}{\partial t_1} + P_c \alpha_1 \frac{\partial Q_1(t_1)}{\partial t_1}$$

$$- \left\{ \sum_{j=1}^{n} \left[ P_c d_1 \alpha_1 W_{ij}(t_1) Q_i(t_1) [1 - \beta_{ij}] + P_c d_1 \alpha_1 W_{ij}(t_1) \frac{\partial Q_i(t_1)}{\partial t_1} [1 - \beta_{ij}] + P_c d_1 \alpha_1 \frac{\partial W_{ij}(t_1)}{\partial t_1} Q_i(t_1) [1 - \beta_{ij}] \right] \right\}$$

(E4)

Dividing both sides by $V_i(t_1)$, and at the optimal harvest age $t_1$:
\[
\sum_{i=0}^{n-1} \frac{P_{ij}(t_1) W_{t1}(t_1)}{V_1(t_1)} + \sum_{i=0}^{n-1} \frac{P_{ij}(t_1) \dot{W}_{t1}(t_1) Q_1(t_1)}{V_1(t_1)} + \sum_{i=0}^{n-1} \frac{P_{ij}(t_1) \dot{W}_{t1}(t_1) Q_1(t_1)}{V_1(t_1)} + \frac{P_i \dot{Q}(t_1)}{V_1(t_1)} + \frac{P_i \dot{Q}(t_1)}{V_1(t_1)}
\]

\[
\left( \frac{u}{u + 1} \right) = r_1
\]

Eq. (E5) represents Pressler's indicator rate formula, with \(u = (V_1(t_1) - E_1(t_1))/LEV_2\).

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