For. Sci. 60(1):43–56
http://dx.doi.org/10.5849/forsci.12-074
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biometrics

Individual Tree Diameter, Height, and Volume Functions for Longleaf Pine

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Currently, little information is available to estimate individual tree attributes for longleaf pine (*Pinus palustris* Mill.), an important tree species of the southeastern United States. The majority of available models are local, relying on stem diameter outside bark at breast height (dbh, cm) and not including stand-level parameters. We developed a set of individual tree equations to predict tree height (H, m), stem diameter inside bark at 1.37 m height (dbh_{IB}, cm), stem volume outside bark (V_{OB}, m^3), and stem volume inside bark (V_{IB}, m^3), as well as functions to determine merchantable stem volume ratio (both outside and inside bark) from the stump to any top diameter. Local and general models are presented for each tree attribute. General models included stand-level parameters such as age, site index, dominant height, basal area, and tree density. The user should decide which model type to use, depending on data availability and level of accuracy desired. To our knowledge, this is the first comprehensive individual tree-level set of equations reported for longleaf pine trees, including local and general models, which can be applied to longleaf pine trees over a large geographical area and across a wide range of ages and stand characteristics. The system presented here provides important new tools for supporting future longleaf pine management decisions.

Keywords: *Pinus palustris*, individual-tree functions, general models, stand variables, merchantable stem volume ratio

Before European settlement, longleaf pine dominated forests in the southeastern United States, occupying about 36 million ha (Frost 1993). Only about 1.2 million ha of longleaf pine stands currently exist (Frost 2006). These remaining longleaf stands extend along the Gulf and Atlantic Coastal Plains from Virginia south into central Florida and north into the Piedmont and mountains of northern Alabama and Georgia. In recent years, various organizations have begun promoting longleaf plantation establishment and sustainable management of existing natural forests, increasing the need for accurate tools to quantify stocks, yield, and dynamics of longleaf pine forests.

Accurate estimates of tree height (H, m), stem diameter outside bark at 1.37 m height (dbh, cm), and stem volume (V, m^3) are central to our ability to understand and predict forest stocks and dynamics. Measures of H and V are needed for estimating site productivity, stand vertical structure, and stand- and tree-level growth and yield (Staudhammer and LeMay 2000, Temesgen et al. 2007, Weiskittel et al. 2011). Bark thickness (bt, cm) and therefore diameter inside bark (dbh_{IB}, cm) and stem volume inside bark (V_{IB}, m^3) are also important tree attributes, useful for bark and wood production quantification (Feduccia and Mann 1975, Tiarks and Haywood 1992). Functions to estimate merchantable stem volume from the stump to any top diameter are useful tools to estimate volume breakdown functions when threshold merchantable limits are known (Burkhart 1977, Amateis et al. 1986).

Often local functions to estimate H, bt, and dbh_{IB} rely on dbh as the explanatory variable (Weiskittel et al. 2011), and functions to estimate V also rely on dbh and H as explanatory variables (Harrison and Borders 1996). These models are widely used but are limited to certain stand characteristics, particularly those from which the data originate. However, inclusion of additional stand variables in these models related to stand age, density, and/or productivity often improves the relationships, resulting in general models that provide more accurate predictions (Larsen and Hann 1987, Huang and Titus 1994, Staudhammer and LeMay 2000, Leduc and Goelz 2009, 2010). In addition, general models allow for better conditions for inter- and extrapolation, and they can provide a sound biological interpretation of the relationships under study.

Few local models that predict H, dbh_{IB}, and V in longleaf pine trees have been produced (Baldwin and Saucier 1983, Farrar 1987,

Manuscript received June 25, 2012; accepted January 14, 2013; published online February 28, 2013.

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Acknowledgments: *This research was supported by the US Department of Defense, through the Strategic Environmental Research and Development Program. The authors acknowledge the USDA Forest Service, Southern Research Station for their assistance and for providing the long-term data sets. Special thanks go to Mr. Tom Stokes, Ms. Ann Huyler, Mr. Justin Rathal, and Mr. Jake Blackstock for their help with field data collection.*

Table 1. Summary of individual tree- and stand-level characteristics for planted longleaf pine in Western Gulf Coastal Plain United States.

	Model development data set ($n = 199$)				Model evaluation data set ($n = 30$)				Model evaluation data set (Fort Benning) ($n = 20$)			
Variable	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Age	37.6	15.6	4	73	39.4	16.8	8	73	17.9	14.9	5.0	87.0
dbh	24.6	10.3	3.6	61	25.1	10.2	3.6	57.4	13.0	12.3	0.6	57.4
H	19.6	6.1	2.4	33.3	20	6.2	3	33.9	9.8	7.6	1.5	32.4
dbh_{IB}	15.8	7.9	2.3	44.5	17.9	8.3	3.3	39.9	NA	NA	NA	NA
bt	13.8	5.3	2.5	40.6	15.2	5	5.1	34.3	NA	NA	NA	NA
V_{OB}	0.5	0.4	0	2.8	0.5	0.4	0	2.1	0.8	1.0	0.0	2.9
N	453	170	175	934	626	448	99	2,145	1,396	589	50	2,150
BA	14	6.8	3.8	33.1	25.7	12.3	0.4	57.6	14.3	9.0	0.4	24.2
SDI	294	109	97	589	504	230	22	1,160	376	235	19	642
H_{dom}	19.8	4.3	10.8	27.6	22.1	6	2.4	32.2	10.5	5.2	3.1	32.4
SI	28.6	2.7	19.6	34	25.4	1.8	21.4	29.2	25.4	4.1	18.2	33.9

Min, minimum; Max, maximum; SD, standard deviation; NA, not applicable.

Quicke and Meldahl 1992), and only one general model to predict H has been reported (Leduc and Goelz 2010). Therefore, the objective of this study was to develop a set of local and general models to predict H, dbh_{IB}, stem volume outside bark (V_{OB}, m^3), and V_{IB} as well as functions to determine merchantable stem volume ratio (R; both outside and inside bark) from the stump to any top diameter. To our knowledge, this is the first comprehensive individual tree-level set of equations for longleaf pine trees that includes local and general models that can be applied to longleaf pine trees over a large geographical area and across a wide range of ages and stand characteristics. Functions to estimate V_{OB}, V_{IB}, and R were used to determine stand-level parameters to develop a comprehensive stand-level growth-and-yield model for the species (Gonzalez-Benecke et al. 2012).

Materials and Methods

Data Description

The data set used to estimate the parameters for individual tree equations for longleaf pine originates from 229 permanent plots measured and maintained by the USDA Forest Service Laboratory at Pineville, Louisiana (Goelz and Leduc 2001). The data were collected from permanent operational plots in a combination of seven studies exploring the effects of spacing and thinning on longleaf pine plantations distributed throughout the Western Gulf Coastal Plain, United States, from Santa Rosa County in Florida to Jasper County in Texas and represent the current range of longleaf pine in the Western Gulf Coastal Plain (Goelz and Leduc 2001).

On 25,695 trees, dbh and H were measured to the nearest 2.54 mm and 3.05 cm, respectively. Tree stem V_{OB} was determined from 9,690 standing trees by measuring diameter outside bark and height to the diameter at 5.08-cm diameter taper steps along the bole from the stump to the point on the stem where diameter was 5.08 cm. On a subset of 2,484 trees, bt was measured at 1.37 m height in two opposite directions, and the average was used for diameter inside bark determinations. Associated with the individual tree-level assessments, basal area (BA, m^2 ha^{-1}), number of trees per ha (N, ha^{-1}), mean dominant height (H_{dom}, m), defined as the mean of the top 25th percentile tree height, and site index (SI, m), defined as the H_{dom} at a reference age of 50 years, were determined for each plot. SI was not directly determined in 79 plots (those plots were not measured at age 50 years); here, SI was predicted using the equation reported by Gonzalez-Benecke et al. (2012). For all trees with H measurement, a form factor F = H/dbh (m cm^{-1}) was calculated.

To eliminate broken and malformed individuals, trees with F less than 0.54 m cm^{-1} and greater than 13.5 m cm^{-1} were excluded from further analyses. Trees with H less than 2.2 m and dbh less than 3 cm were also discarded from the data set for diameter-height analysis. From the complete data set, 30 plots were randomly selected and removed to use for model evaluation, and the rest (i.e., 237 plots) were used for model fitting. The model evaluation data set contained 3,163 trees for the dbh versus H relationship, 1,254 trees for V modeling, and 264 trees for dbh_{IB} modeling. A second independent source of evaluation data (described below) included stands planted outside the geographic range of the data used for model development. Details of tree and stand characteristics of the fitting data set and of both evaluation data sets are shown in Table 1. General relationships between dbh and H and V_{OB} are shown in Figure 1.

Model Description

The following model proposed by Parresol (1992) was used to estimate total tree height

$$(H - 1.37) = e^{a_1 + a_2 \cdot dbh^{a_3}} + \varepsilon_1 \tag{1}$$

where a_1 to a_3 are curve-fit parameters and ε_1 is the error term, with $\varepsilon_1 \sim N(0, \sigma_1^2)$. This tree-height static local model is commonly fitted considering a fixed power a_3 of -1.0 (Quicke and Meldahl 1992), but in some cases other values can produce better fits (Curtis 1967, Larsen and Hann 1987, Wang and Hann 1988, Zhao et al. 2006).

In addition to dbh, several stand-level variables were included as covariates in the above model to improve the local height-diameter equation, resulting in a general height-diameter equation. The variables considered corresponded to Age, N, BA, H_{dom}, stand density index (SDI, ha^{-1}), and quadratic mean diameter (D_q, cm). These variables were selected because they represent different aspects of the stand, such as stocking, productivity, and competition, which could affect the height-diameter relationships. A model was fitted considering all potential variables, and a simplified version was also evaluated that did not consider SI or H_{dom}, because these are not always available. Similar to the method of Crecente-Campo et al. (2010), to test which stand-level variables should be included in the final general model, a logarithm transformation was performed, and a stepwise procedure was used on the resulting linear model with a threshold significance value of 0.15 as variable selection criteria, and the

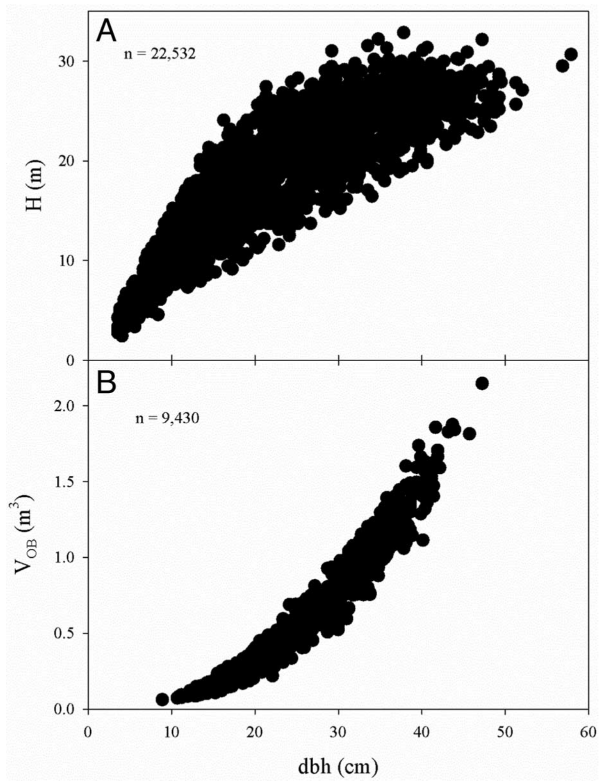

Figure 1. Relationships between dbh and H (A) and V_{OB} (B) using the model-fitting data set.

variance inflation factor (VIF) was monitored to detect multicollinearity between explanatory variables. All variables included in the model with VIF > 5 were discarded, as suggested by Neter et al. (1996). The final nonlinear forms of the models finally selected to estimate H were

$$(\mathrm{H} - 1.37) = e^{(a_1 + a_2 \cdot \mathrm{dbh}^{a_3} + \mathrm{Age}^{a_4} + \mathrm{BA}^{a_5})} + \varepsilon_2 \quad (2)$$

$$(\mathrm{H} - 1.37) = e^{(a_1 + a_2 \cdot \mathrm{dbh}^{a_3} + \mathrm{Age}^{a_4} + \mathrm{BA}^{a_5} + \mathrm{SI}^{a_6})} + \varepsilon_3 \quad (3)$$

where Age is the stand age (years), BA is the basal area ($\mathrm{m}^2\ \mathrm{ha}^{-1}$), SI is the site index at age 50 years (m), a_1 to a_6 are curve-fit parameters, and ε_2 and ε_3 are the error terms, with $\varepsilon_i \sim N(0, \sigma_i^2)$.

Functions to estimate $\mathrm{dbh_{IB}}$ and V_{OB} were fitted using dbh as an independent variable in the following models

$$\mathrm{dbh_{IB}} = b_1 + b_2 \cdot (\mathrm{dbh}) + \varepsilon_4 \quad (4)$$

$$V_{OB} = c_1 \cdot \mathrm{dbh}^{c_2} + \varepsilon_5 \quad (5)$$

If H is known, an alternative model to determine V_{OB} was also fitted

$$V_{OB} = d_1 \cdot \mathrm{dbh}^{d_2} \cdot \mathrm{H}^{d_3} + \varepsilon_6 \quad (6)$$

where b_1, b_2, b_3, c_1, c_2, c_3, d_1, d_2, and d_3, are curve-fit parameters, and ε_4, ε_5, and ε_6 are the error terms, with $\varepsilon_i \sim N(0, \sigma_i^2)$.

Following the same procedure of linear log transformation and variable selection criteria used for dbh-H models, general models that include stand-level variables were also fitted for Equations 4–6. The model finally selected to estimate $\mathrm{dbh_{IB}}$ was

$$\mathrm{dbh_{IB}} = b_1 + b_2 \cdot \mathrm{dbh} + b_3 \cdot \mathrm{Age} + \varepsilon_7 \quad (7)$$

If $\mathrm{H_{dom}}$ and SI are also known, the alternative general model finally selected was

$$\mathrm{dbh_{IB}} = b_1 + b_2 \cdot \mathrm{dbh} + b_3 \cdot \mathrm{Age} + b_4 \cdot \mathrm{BA} + b_5 \cdot \mathrm{S1} + \varepsilon_8 \quad (8)$$

where b_1 to b_5 are curve-fit parameters and ε_7 and ε_8 are the error terms, with $\varepsilon_i \sim N(0, \sigma_i^2)$.

Because diameter inside bark was not measured directly at each step along the bole where diameter outside bark was measured, we used the equation reported by Cao and Pepper (1986) that predicts diameter inside bark at any stem height, from outside diameter, outside and inside dbh, relative height, and total height for planted longleaf pines. Stem volume inside bark was determined in the same way as V_{OB}. Similarly to V_{OB}, local and general functions to estimate V_{IB} were fitted using dbh and dbh and H as independent variables.

The models finally selected to estimate V (both outside and inside bark) were

$$V_{OB} \text{ or } V_{IB} = c_1 \cdot (\mathrm{dbh}^{c_2}) \cdot (\mathrm{Age}^{c_3}) \cdot (\mathrm{N}^{c_4}) \cdot (\mathrm{BA}^{c_5}) + \varepsilon_9 \quad (9)$$

$$V_{OB} \text{ or } V_{IB} = d_1 \cdot (\mathrm{dbh}^{d_2}) \cdot (\mathrm{H}^{d_3}) \cdot (\mathrm{Age}^{d_4}) \cdot (\mathrm{BA}^{d_5}) + \varepsilon_{10} \quad (10)$$

If $\mathrm{H_{dom}}$ and SI are also known, the alternative general models finally selected were

$$V_{OB} \text{ or } V_{IB} = c_1 \cdot (\mathrm{dbh}^{c_2}) \cdot (\mathrm{N}^{c_3}) \cdot (\mathrm{BA}^{c_4}) \cdot (\mathrm{H_{dom}}^{c_5}) \cdot (\mathrm{SI}^{c_6}) + \varepsilon_{11} \quad (11)$$

$$V_{OB} \text{ or } V_{IB} = d_1 \cdot (\mathrm{dbh}^{d_2}) \cdot (\mathrm{H}^{d_3}) \cdot (\mathrm{N}^{d_4}) \cdot (\mathrm{BA}^{d_5}) \cdot (\mathrm{H_{dom}}^{d_6}) + \varepsilon_{12} \quad (12)$$

where c_1 to c_6 and d_1 to d_6 are curve-fit parameters and ε_9 to ε_{12} are the error terms, with $\varepsilon_i \sim N(0, \sigma_i^2)$.

A model to estimate merchantable stem volume (both outside and inside bark) from the stump to any top diameter was fitted following the method of Burkhart (1977), in which a function that predicts the ratio of merchantable stem volume (both outside and inside bark) divided by total stem volume was determined as follows

$$\mathrm{R} = 1 + e_1 \cdot \left(\frac{d_t^{e_2}}{\mathrm{dbh}^{e_3}}\right) + \varepsilon_{13} \quad (13)$$

where R is the ratio between merchantable stem volume (inside or outside bark) to top diameter outside bark (d_t, cm) and total stem volume up to a 5.08-cm top limit, e_1, e_2, and e_3 are curve-fit parameters, and ε_{13} is the error term, with $\varepsilon_{13} \sim N(0, \sigma_{13}^2)$. Average stump height was considered to be 20 cm.

Model Evaluation

All statistical analyses were performed with SAS 9.3 (SAS, Inc., Cary, NC). The predictive ability of the local and general equations was evaluated by comparing predictions with data from trees in the evaluation data set. The models that estimate H (Equations 1, 4, and 5) and V_{OB} (Equations 7, 8, 11, 12, 13, and 14) were also evaluated against an independent data set consisting of five stands from Fort Benning, Georgia. The stands had ages of 5, 11, 21, 64, and 87 years. In each stand, four 0.04-ha inventory plots were measured, recording H and dbh in all trees. In a subset of 11 trees (5 from the 21-year-old stand, 3 from the 64-year-old stand, and 3 from the 87-year-old stand), stem volume over bark was directly measured by

Table 2. Parameter estimates and fit statistics of the Western Gulf Coastal Plain United States planted longleaf pine tree equations.

	Model	Parameter	Parameter estimate	SE	n	R^2	RMSE	CV
H1	$H - 1.37 = a_1 + a_2 \cdot dbh^{a_3}$	a_1	3.773937	0.019028	22,532	0.977	2.92	14.88
		a_2	−7.844121	0.194658				
		a_3	−0.710479	0.015122				
$dbh_{IB}1$	$dbh_{IB} = b_1 + b_2 \cdot dbh$	b_1	−0.869346	0.027399	2,173	0.995	0.53	3.30
		b_2	0.897180	0.001320				
$V_{OB}1$	$V_{OB} = c_1 \cdot dbh^{c_2}$	c_1	0.000457	0.000009	9,430	0.986	0.08	14.94
		c_2	2.187608	0.005386				
$V_{IB}1$	$V_{IB} = c_1 \cdot dbh^{c_2}$	c_1	0.000264	0.000005	9,430	0.986	0.06	11.18
		c_2	2.260839	0.005491				
$V_{OB}4$	$V_{OB} = d_1 \cdot dbh^{d_2} \cdot H^{d_3}$	d_1	0.000054	0.000001	9,430	0.996	0.04	8.04
		d_2	1.842136	0.003652				
		d_3	1.070207	0.007229				
$V_{IB}4$	$V_{IB} = d_1 \cdot dbh^{d_2} \cdot H^{d_3}$	d_1	0.000031	0.000001	9,430	0.996	0.03	6.13
		d_3	1.917270	0.003781				
		d_2	1.072289	0.007467				
R_{OB}	$R_{OB} = 1 - e_1 \cdot (d_t^{e_2}/dbh^{e_3})$	e_1	0.532609	0.003100	35,571	0.996	0.04	6.28
		e_2	3.997480	0.007351				
		e_3	3.808041	0.007641				
R_{IB}	$R_{IB} = 1 - e_1 \cdot (d_t^{e_2}/dbh^{e_3})$	e_1	0.551688	0.003183	35,571	0.995	0.05	6.20
		e_2	3.975042	0.007266				
		e_3	3.792207	0.007554				

R_{OB}, ratio between merchantable stem volume outside bark to top diameter d_t divided by total stem volume outside bark up to 5.08-cm diameter limit outside bark; R_{IB}, ratio between merchantable stem volume inside bark to top diameter d_t divided by total stem volume inside bark up to 5.08 cm diameter limit outside bark. For all parameter estimates: $P < 0.001$.

destructive harvesting of the trees and measuring bole diameter over bark at 2-m steps from stump to a minimum diameter of 5 cm. This measurement is part of biomass sampling for a project on developing tools for ecological forestry and carbon management in longleaf pine (Center for Longleaf Pine Ecosystems 2012). The three younger stands were planted, and the two older stands were naturally regenerated.

Four measures of accuracy were used to evaluate the goodness of fit between observed and predicted values for each variable based on the model evaluation data set: (1) mean absolute error (MAE); (2) root mean square error (RMSE); (3) mean bias error (Bias); and (4) coefficient of determination (R^2; Fox 1981, Loague and Green 1991, Kobayashi and Salam 2000).

Using the same data set for model evaluation, the equations were also compared against other models reported in the literature for longleaf pine trees. These corresponded to the following: (1) diameter-height equations reported by Quicke and Meldahl (1992) and Leduc and Goelz (2010); (2) diameter inside bark at breast height equation reported by Farrar (1987); (3) total stem volume equations reported by Baldwin and Saucier (1983); and (4) merchantable stem volume from the stump to any top diameter equation reported by Saucier et al. (1981).

Results

The model parameter estimates for the planted longleaf pine trees growing in the Western Gulf Coastal Plain United States are reported in Tables 2 and 3. Table 2 includes only local equations (i.e., do not consider stand-level variables). All parameter estimates were significant at $P < 0.001$.

Model Fitting

The model that estimates H using dbh as the only dependent variable (H1) has a coefficient of variation (CV; RMSE as a percentage of observed mean value) of 14.9% (Table 2). When stand parameters Age, N, BA, H_{dom}, and SI were included in the model (H3), N and H_{dom} were not significant into the final model that minimized the sum of squares of Equation 2, having a CV of 7.6% (Table 3). As an alternative model, we fitted the equation without SI and H_{dom} (H2): this model had a RMSE 1.36% larger than the RMSE of the model that included SI but 40% smaller than the RMSE of the model that only used dbh as predictor. The parameters Age, BA, and SI had a positive effect on H (positive value of parameter estimates): as the value of those parameters increased, the height of the tree was larger for any given tree dbh. In all cases, multicollinearity between explanatory variables was small (VIF < 5).

The model that estimates dbh_{IB} as a function of dbh ($dbh_{IB}1$) had CV, RMSE, and R^2 of about 3.53, 0.53, and 0.995%, respectively. When stand parameters Age, N, BA, H_{dom}, and SI were included in the model ($dbh_{IB}3$), Age, BA, H_{dom} and SI were significant, but because H_{dom} presented a VIF of 25.7, it was dropped from the final model (data not shown). The final general model to estimate dbh_{IB} only slightly improved the fit, having RMSE and CV of about 3.2 and 0.51%, respectively. An alternative model was fitted, assuming that H_{dom} and SI are unknown. This optional model ($dbh_{IB}2$) presented similar CV, RMSE, and R^2 than the whole model described previously. In all cases, the multicollinearity between explanatory variables was small (VIF < 5).

Two different local models were fitted to estimate V (both outside and inside bark): using dbh (Equation 5) or using dbh and H (Equation 6) as independent variables. Including H highly improved the fit of the model: the former model that used only dbh ($V_{OB}1$) had a CV and RMSE of about 14.9 and 0.08%, respectively, whereas the model that used dbh and H ($V_{OB}4$) had a CV and RMSE of about 8.1 and 0.04% (Table 2). For the models that depended on dbh, when stand parameters Age, N, BA, H_{dom}, and SI were included, all variables were significant in the final model, but because Age produced a VIF of 16.4 (data not shown), it was dropped from the final model. The final general model, which included N, BA, H_{dom}, and SI as explanatory variables ($V_{OB}3$), had CV and RMSE of about 10.4 and 0.05%, respectively (Table 3). An alternative model was fitted, assuming that H_{dom} and SI are unknown: the resulting model ($V_{OB}2$) was dependent, besides on dbh, on Age, N, and BA and had both a CV and RMSE 14% smaller

Table 3. Parameter estimates and fit statistics of the Western Gulf Coastal Plain United States planted longleaf pine trees equations including stand variables.

	Model	Parameter	Parameter estimate	SE	n	VIF	R^2	RMSE	CV
H2	$H - 1.37 = e^{(a_1 + a_2 \cdot \mathrm{dbh}^{a_3} + \mathrm{Age}^{a_4} + \mathrm{BA}^{a_5})}$	a_1	0.059425	0.009450	22,532		0.992	1.762	8.97
		a_2	−10.803775	0.298591		2.55			
		a_3	−1.127503	0.014269		4.95			
		a_4	0.150532	0.000942		3.43			
		a_5	0.121239	0.000832		2.61			
H3	$H - 1.37 = e^{(a_1 + a_2 \cdot \mathrm{dbh}^{a_3} + \mathrm{Age}^{a_4} + \mathrm{BA}^{a_5} + \mathrm{SI}^{a_6})}$	a_1	−2.573981	0.015599	22,532		0.994	1.495	7.61
		a_2	−13.977064	0.350413		2.42			
		a_3	−1.266507	0.012507		3.49			
		a_4	0.185690	0.000724		1			
		a_5	0.078576	0.001010		1.58			
		a_6	0.283800	0.001563		1.46			
$\mathrm{dbh}_{IB}2$	$\mathrm{dbh}_{IB} = b_1 + b_2 \cdot \mathrm{dbh} + b_3 \cdot \mathrm{Age}$	b_1	−1.089962	0.034429	2,173		0.996	0.519	3.23
		b_2	0.886879	0.001639		1.61			
		b_3	0.014879	0.001460		1.61			
$\mathrm{dbh}_{IB}3$	$\mathrm{dbh}_{IB} = b_1 + b_2 \cdot \mathrm{dbh} + b_3 \cdot \mathrm{Age} + b_4 \cdot \mathrm{BA} + b_5 \cdot \mathrm{SI}$	b_1	−2.150585	0.170537	2,173		0.996	0.514	3.20
		b_2	0.886310	0.001642		1.65			
		b_3	0.022423	0.002441		4.59			
		b_4	−0.005599	0.003249		3.61			
		b_5	0.032070	0.005073		1.36			
$V_{OB}2$	$V_{OB} = c_1 \cdot (\mathrm{dbh}^{c_2}) \cdot (\mathrm{Age}^{c_3}) \cdot (\mathrm{N}^{c_4}) \cdot (\mathrm{BA}^{c_5})$	c_1	0.000078	0.000003	9,430		0.990	0.066	12.78
		c_2	2.099555	0.006652		1.86			
		c_3	0.540248	0.009501		3.92			
		c_4	0.085189	0.004088		2.89			
		c_5	−0.145425	0.004398		4.80			
$V_{OB}3$	$V_{OB} = c_1 \cdot (\mathrm{dbh}^{c_2}) \cdot (\mathrm{N}^{c_3}) \cdot (\mathrm{BA}^{c_3}) \cdot (\mathrm{H_{dom}}^{c_5}) \cdot (\mathrm{SI}^{c_6})$	c_1	0.000031	0.000001	9,430		0.993	0.054	10.40
		c_2	2.078588	0.005352		1.84			
		c_3	0.065959	0.003444		3.17			
		c_4	−0.108270	0.003745		4.52			
		c_5	1.085691	0.013359		2.88			
		c_6	−0.127886	0.015951		1.71			
$V_{IB}2$	$V_{IB} = c_1 \cdot (\mathrm{dbh}^{c_2}) \cdot (\mathrm{Age}^{c_3}) \cdot (\mathrm{N}^{c_4}) \cdot (\mathrm{BA}^{c_5})$	c_1	0.000045	0.000002	9,430		0.990	0.050	9.61
		c_2	2.173753	0.006798		1.86			
		c_3	0.541315	0.009676		3.92			
		c_4	0.084559	0.004127		2.89			
		c_5	−0.145635	0.004456		4.80			
$V_{IB}3$	$V_{IB} = c_1 \cdot (\mathrm{dbh}^{c_2}) \cdot (\mathrm{N}^{c_3}) \cdot (\mathrm{BA}^{c_3}) \cdot (\mathrm{H_{dom}}^{c_5}) \cdot (\mathrm{SI}^{c_6})$	c_1	0.000018	0.000001	9,430		0.993	0.04	7.85
		c_2	2.151768	0.005495		1.84			
		c_3	0.065305	0.003492		3.17			
		c_4	−0.108915	0.003819		4.52			
		c_5	1.095234	0.013734		2.88			
		c_6	−0.136949	0.016283		1.71			
$V_{OB}5$	$V_{OB} = d_1 \cdot (\mathrm{dbh}^{d_2}) \cdot (\mathrm{H}^{d_3}) \cdot (\mathrm{Age}^{d_4}) \cdot (\mathrm{BA}^{d_5})$	d_1	0.000046	0.000001	9,430		0.996	0.04	7.82
		d_2	1.856293	0.004046		1.74			
		d_3	1.012445	0.007939		2.31			
		d_4	0.101381	0.005997					
		d_5	−0.030956	0.001454		1.74			
$V_{OB}6$	$V_{OB} = d_1 \cdot (\mathrm{dbh}^{d_2}) \cdot (\mathrm{H}^{d_3}) \cdot (\mathrm{N}^{d_4}) \cdot (\mathrm{BA}^{d_5}) \cdot (\mathrm{H_{dom}}^{d_6})$	d_1	0.000056	0.000002	9,430		0.996	0.04	7.85
		d_2	1.851051	0.004908		1.73			
		d_3	1.046938	0.012733		2.23			
		d_4	−0.021418	0.002519		1.06			
		d_5	−0.003044	0.002522					
		d_6	0.041285	0.014362		1.51			
$V_{IB}5$	$V_{IB} = d_1 \cdot (\mathrm{dbh}^{d_2}) \cdot (\mathrm{H}^{d_3}) \cdot (\mathrm{Age}^{d_4}) \cdot (\mathrm{BA}^{c_5})$	d_1	0.000026	0.000001	9,430		0.996	0.03	5.98
		d_2	1.931795	0.004190		1.74			
		d_3	0.105677	0.006205		2.31			
		d_4	−0.032210	0.001500					
		d_5	1.013361	0.008168		1.74			
$V_{IB}6$	$V_{IB} = d_1 \cdot (\mathrm{dbh}^{d_2}) \cdot (\mathrm{H}^{d_3}) \cdot (\mathrm{N}^{d_4}) \cdot (\mathrm{BA}^{d_5}) \cdot (\mathrm{H_{dom}}^{d_6})$	d_1	0.000032	0.000001	9,430		0.996	0.03	6.00
		d_2	1.926111	0.005073		1.73			
		d_3	1.048862	0.013118		2.23			
		d_4	−0.022558	0.002589		1.06			
		d_5	−0.002865	0.002605					
		d_6	0.042822	0.014821		1.15			

For all parameter estimates: $P < 0.001$.

than that of the reduced model. For $V_{OB}2$, the parameters for Age and N had a positive effect on V (positive value of parameter estimate): trees of the same dbh will have a greater V_{OB} if they are older or they are growing in stands with larger tree density. Only the parameter for BA had a negative sign: trees of the same dbh, age, and growing in stands with the same N will have smaller V_{OB} if they are growing in stands with larger BA. This effect should be related with changes in tapering and crown length in those stands.

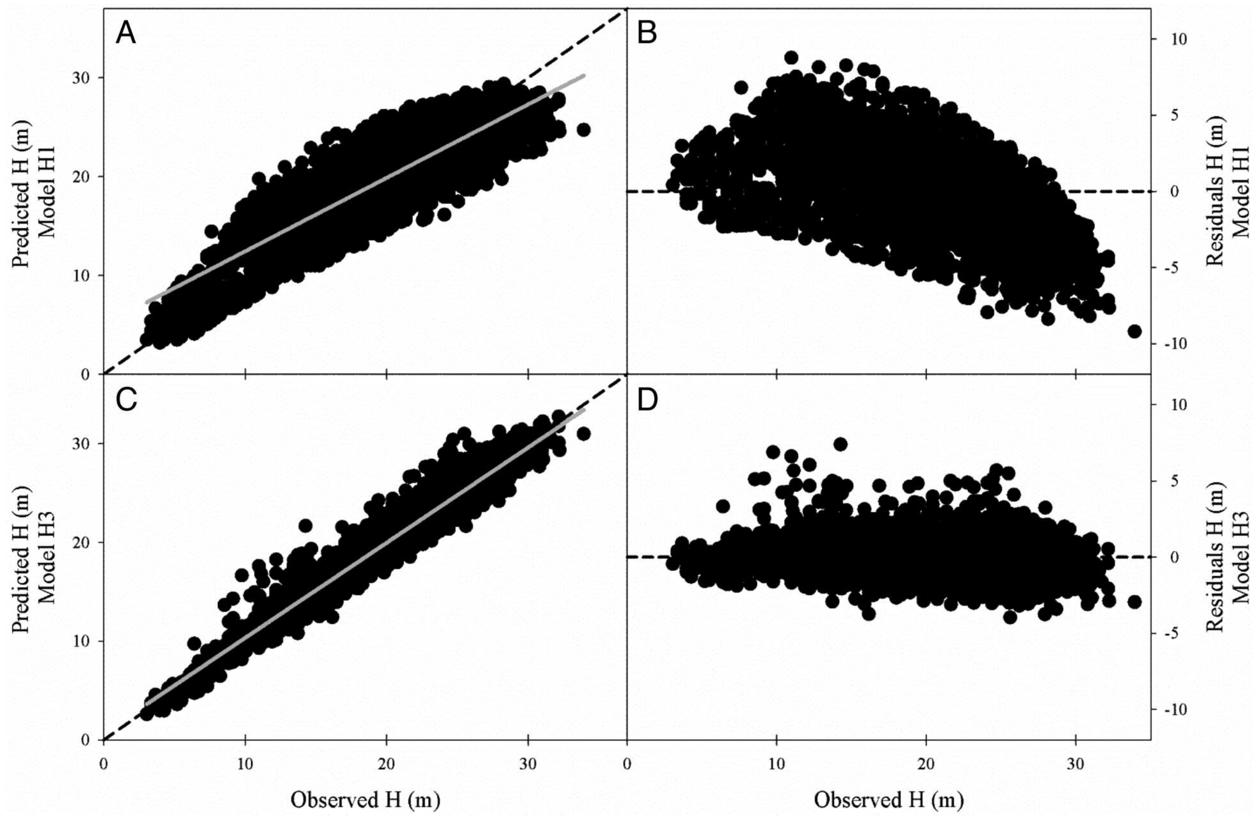

Figure 2. Examples of evaluation of total tree height (H) models. Observed versus predicted (simulated) values using the local model (model H1, that only uses dbh as explanatory variable) (A) and the general model (model H3, with dbh, Age, BA, N, and SI) (C) and residuals (predicted − observed) versus observed values using model H1 (B) and model H3 (D). The gray line in the left panels represents linear fit between observed and predicted values.

For the model to estimate V that depended on dbh and H, stand parameters had little effect on model fitting even if they were statistically significant. The model that minimized the sum of squares included, besides dbh and H, Age, N, H_{dom}, and SI, but H_{dom} was dropped because of the high VIF (18.5, respectively, data not shown); however, this new model had little affect on model fit, reducing both CV and RMSE by less than 1% (V_{OB}6). The parameter N had a negative value in the V_{OB}6 model, implying that trees of the same dbh and H will have a smaller V if they are growing in stands with larger N. This effect should be related to changes in tapering and crown length in those stands. An alternative model was also fitted, assuming that H_{dom} and SI are not known (V_{OB}5). The resulting model was dependent on Age and BA in addition to dbh and H. For V_{IB} models, the fit was always slightly better than that for V_{OB} models with the same set of predicting variables. In all cases, VIF < 5 (Table 3).

The models that estimate the ratio of merchantable stem volume (both outside and inside bark) to top diameter outside bark d_t divided by total stem volume up to 5.08-cm diameter limit outside bark (Equation 13) had a CV, RMSE, and R^2 of about 6.2, 0.048, and 0.996%, respectively (Table 2).

Model Validation

The relationship between predicted and observed values of H using the general model, which only depended on dbh (model H1, Figure 2A and B), showed a tendency to underestimate the results for trees with H larger than about 25 m. When stand parameters Age, BA, and SI were included in the model, the relationship between observed and predicted values improved considerably (model H3, Figure 2C and D), and there was no noticeable trend in residuals against observed values. All model performance tests showed that H estimations improved their agreement with measured values when stand parameters were included in the general model (Table 4). For example, MAE and RMSE were reduced from 11.9 and 14.6% (model H1) to 4.9 and 6.5% (model H3), respectively, and R^2 was increased from 0.784 to 0.957, respectively.

There was good agreement between predicted and observed values for dbh_{IB} (Figure 3A and C), with no noticeable trend in residuals against observed values (Figure 3B and D). For dbh_{IB} models, there was no model performance improvement when stand parameters were included.

There was good agreement between predicted and observed values for V (Figure 4, only V_{OB} showed), with no noticeable trend in residuals against observed values. For the model that used dbh as an explanatory variable (Figure 4A and B), when stand parameters were included, the final local model, showed a better agreement (Figure 4C) with reduced dispersion of residuals (Figure 4D). On the other hand, the model that used only dbh and H as explanatory variables (Figure 4E and F) showed better agreement than the model that used only dbh (Figure 4G and H). Performance tests showed that V estimations that use dbh as explanatory variable were improved when stand parameters were included in the general model (Table 4). For example, Bias was reduced from 1.4% (model V_{OB}1) to 0.6% (model V_{OB}3) underestimations, and MAE and RMSE were

Table 4. Summary of model evaluation statistics for H, dbh_{IB}, V_{OB}, V_{IB}, R_{OB}, and R_{IB} estimations.

Dependent variable	Model	Explanatory variables	$\bar{O}$	$\bar{P}$	n	MAE	RMSE	Bias	R^2
H	H1	dbh	20.03	19.89	3,163	2.374 (11.9)	2.915 (14.6)	−0.132 (−0.7)	0.784
	H2	dbh, Age, BA	20.03	20.32	3,163	1.459 (7.3)	1.849 (9.2)	0.295 (1.5)	0.915
	H3	dbh, Age, BA, SI	20.03	19.99	3,163	0.99 (4.9)	1.297 (6.5)	−0.033 (−0.2)	0.957
dbh_{IB}	$dbh_{IB}1$	dbh	17.95	17.97	263	0.432 (2.4)	0.551 (3.1)	0.023 (0.1)	0.996
	$dbh_{IB}2$	dbh, Age	17.95	18.01	263	0.397 (2.2)	0.517 (2.9)	0.054 (0.3)	0.996
	$dbh_{IB}3$	dbh, Age, BA, SI	17.95	18.02	263	0.395 (2.2)	0.514 (2.9)	0.067 (0.4)	0.996
V_{OB}	$V_{OB}1$	dbh	0.544	0.536	1,254	0.054 (9.9)	0.076 (14.0)	−0.008 (−1.4)	0.963
	$V_{OB}2$	dbh, Age, N, BA	0.544	0.54	1,254	0.046 (8.5)	0.068 (12.4)	−0.004 (−0.8)	0.97
	$V_{OB}3$	dbh, N, BA, H_{dom}, SI	0.544	0.541	1,254	0.038 (7.0)	0.057 (10.5)	−0.003 (−0.6)	0.979
	$V_{OB}4$	dbh, H	0.544	0.543	1,254	0.029 (5.3)	0.045 (8.2)	−0.001 (−0.2)	0.987
	$V_{OB}5$	dbh, H, Age, BA	0.544	0.542	1,254	0.028 (5.1)	0.042 (7.8)	−0.002 (−0.4)	0.988
	$V_{OB}6$	dbh, H, N, BA, H_{dom}	0.544	0.541	1,254	0.028 (5.2)	0.043 (7.9)	−0.003 (−0.5)	0.988
V_{IB}	$V_{IB}1$	dbh	0.401	0.395	1,254	0.04 (10.0)	0.057 (14.2)	−0.005 (−1.3)	0.964
	$V_{IB}2$	dbh, Age, N, BA	0.401	0.398	1,254	0.034 (8.6)	0.051 (12.7)	−0.003 (−0.7)	0.971
	$V_{IB}3$	dbh, N, BA, H_{dom}, SI	0.401	0.398	1,254	0.029 (7.1)	0.044 (10.9)	−0.002 (−0.6)	0.979
	$V_{IB}4$	dbh, H	0.401	0.399	1,254	0.022 (5.5)	0.034 (8.5)	−0.002 (−0.5)	0.987
	$V_{IB}5$	dbh, H, Age, BA	0.401	0.398	1,254	0.021 (5.3)	0.032 (8.1)	−0.003 (−0.7)	0.988
	$V_{IB}6$	dbh, H, N, BA, H_{dom}	0.401	0.399	1,254	0.022 (5.4)	0.033 (8.2)	−0.002 (−0.5)	0.988
R_{OB} $d_t = 10.16$	R_{OB}	dbh, d_t	0.933	0.927	1,199	0.015 (1.6)	0.033 (3.5)	−0.005 (−0.6)	0.931
R_{IB} $d_t = 10.16$	R_{IB}	dbh, d_t	0.938	0.932	1,199	0.014 (1.5)	0.032 (3.4)	−0.006 (−0.7)	0.931
R_{OB} $d_t = 15.24$	R_{OB}	dbh, d_t	0.836	0.834	1,050	0.028 (3.4)	0.045 (5.3)	−0.003 (−0.3)	0.934
R_{IB} $d_t = 15.24$	R_{IB}	dbh, d_t	0.828	0.826	1,050	0.029 (3.5)	0.045 (5.5)	−0.002 (−0.3)	0.934

R_{OB}, ratio between merchantable stem volume outside bark to top diameter d_t *divided by total stem volume outside bark up to 5.08-cm diameter limit outside bark;* R_{IB}, *ratio between merchantable stem volume inside bark to top diameter* d_t *divided by total stem volume inside bark up to 5.08-cm diameter limit outside bark;* $\bar{O}$, mean observed value; $\bar{P}$, mean predicted value. Values in parentheses are percentage relative to observed mean. MAE, RMSE, and Bias are presented in the same units as the dependent variable.

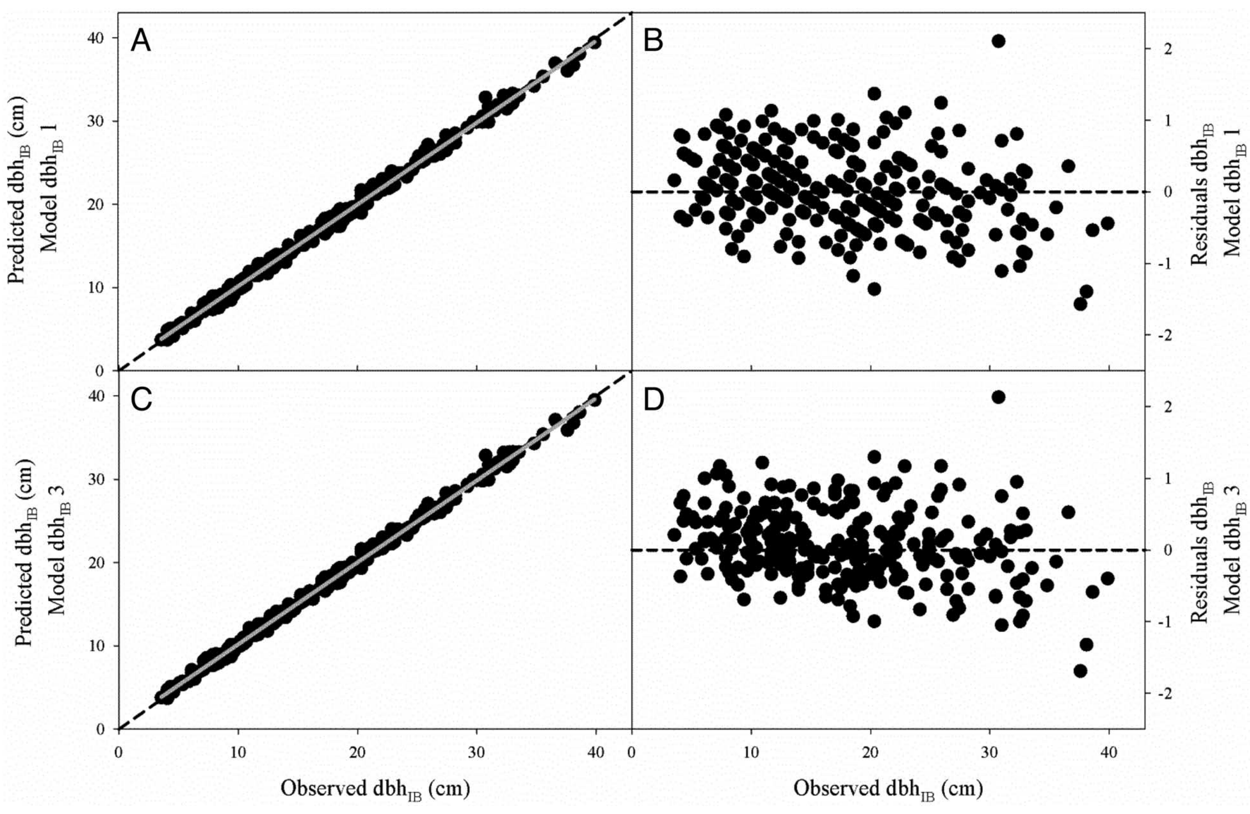

Figure 3. Examples of evaluation of dbh_{IB} models. Observed versus predicted (simulated) values for dbh_{IB} using the local model (model $dbh_{IB}1$, that only uses dbh as explanatory variable) (A) and the general model (model $dbh_{IB}3$, with dbh and H_{dom}) (C) and residuals (predicted − observed) versus observed values using model $dbh_{IB}1$ (B) and model $dbh_{IB}3$ (D). The gray line in the left panels represents linear fit between observed and predicted values.

reduced from 9.9 and 14.0% (model $V_{OB}1$) to 7.0 and 10.5% (model $V_{OB}3$), respectively. In the case of V estimations that use dbh and H as explanatory variables, the improvement in model performance was marginal when stand parameters were included (MAE and RMSE were reduced from 5.3 and 8.2% [model $V_{OB}4$] to 5.2 and 7.9% [model $V_{OB}6$], respectively). Estimated and observed values were highly correlated, with R^2 values greater than 0.96.

For the two examples of d_t used ($d_t = 10.16$ cm, Figure 5A, C,

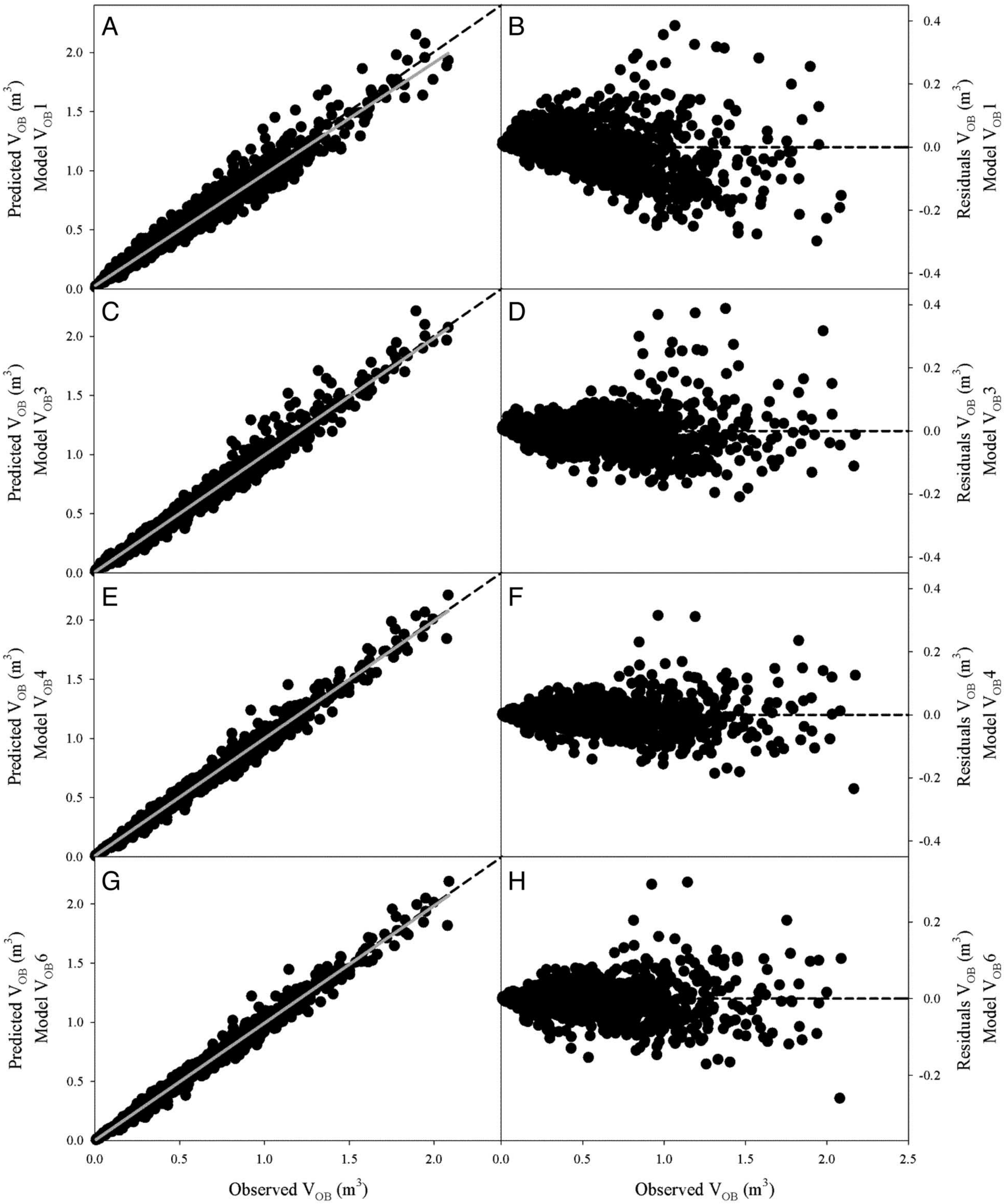

Figure 4. Examples of evaluation of V_{OB} models. Observed versus predicted (simulated) values for V_{OB} using local models (model $V_{OB}1$, that only uses dbh as explanatory variable) (A), model $V_{OB}4$ (that only uses dbh and H as explanatory variable) (E), and general models (model $V_{OB}3$, with dbh, N, H_{dom}, and SI [C]; model $V_{OB}6$, with dbh, H, N, H_{dom}, and SI [G]) and residuals (predicted − observed) versus observed values using model $V_{OB}1$ (B), model $V_{OB}3$ (D), model $V_{OB}4$ (F), and model $V_{OB}6$ (H). The gray line in the left panels represents linear fit between observed and predicted values.

and E; d_t = 15.24 cm, Figure 5B, D, and F), there was good agreement between predicted and observed R_{OB} (Figure 5A and B) and merchantable volume outside bark ($V_{m\text{-}OB}$, Figure 5C and D). There was more data dispersion for small R_{OB} (Figure 5A and B), that correspond with trees with dbh closer to d_t (Figure 5E and F), but the magnitude of that error (m^3) is negligible when $V_{m\text{-}OB}$ is calculated (Figure 5C and D). There was no noticeable trend of residuals with observed values (data not shown). All model

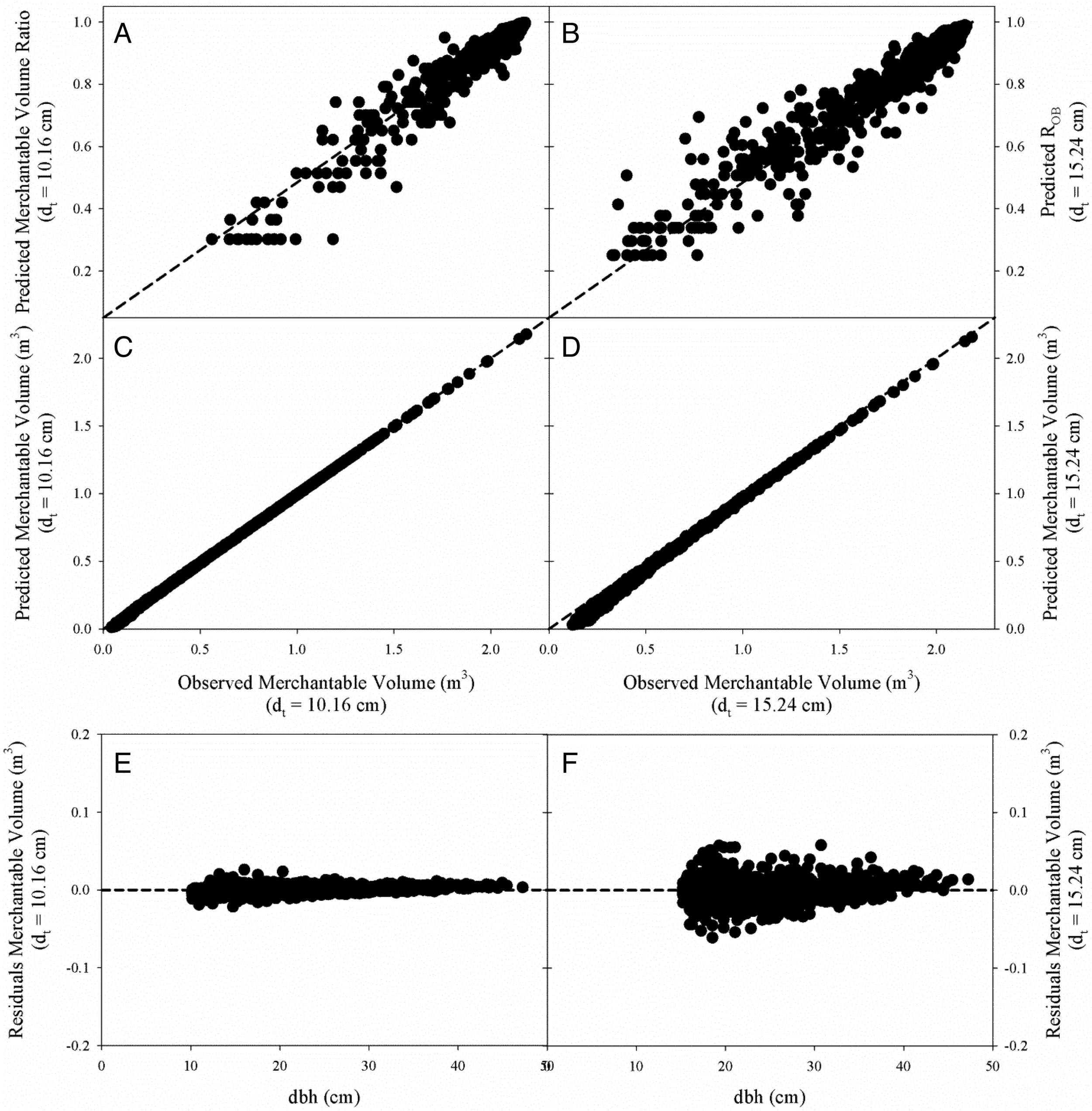

Figure 5. Evaluation of merchantable stem volume from the stump to any top diameter model. Observed versus predicted (simulated) values and residuals (predicted − observed) versus observed values of merchantable volume outside bark ratio (R_{OB}, $m^3 \cdot m^{-3}$) (A and B) and merchantable volume outside bark ($V_{m\text{-}OB}$, m^3) (C and D). Residuals of $V_{m\text{-}OB}$ versus observed dbh (E and F). Two examples of merchantable volume outside bark are shown: using d_t = 10.16 (A, C, and E) and d_t = 15.24 cm (B, D, and F).

performance tests showed that estimations of R (both outside and inside bark) agreed well with measured values (Table 4). For the two d_t values used, MAE and RMSE ranged between 1.5 and 3.5% and 3.4 and 5.5% of the observed values, respectively. The Bias ranged between 0.3 and 0.7% underestimations. Estimated and observed values were highly correlated, with R^2 values greater than 0.93.

Comparison Against Reported Equations for Longleaf Pine

When tested on the data set used for model evaluation, predicted values of the models proposed in this study for H, dbh_{IB}, V_{OB}, and R_{OB} are within the range of variation of the estimations using other published longleaf pine equations. The effects of tree age on H, dbh_{IB}, V_{OB}, and R_{OB} estimations for several models for longleaf pine trees are presented in Figure 6.

Across four age classes (<20, 21–40, 41–60, and 61–73 years), the models reported in this study predicted dbh_{IB} and V_{OB} consistently, with no clear trend to over- or underestimate, with Bias less than 6% and RMSE less than 16% (Figure 6). However, for all models, there was a general trend to reduce RMSE% as trees aged (right panels in Figure 6). For H estimations for trees younger than 20 years old using the model that used only dbh as an explanatory variable (H1, Figure 6A and B), Bias and RMSE averaged approximately 16 and 27%, respectively. For that age range, H estimations with the models reported by Quicke and Meldahl (1992) (H4,

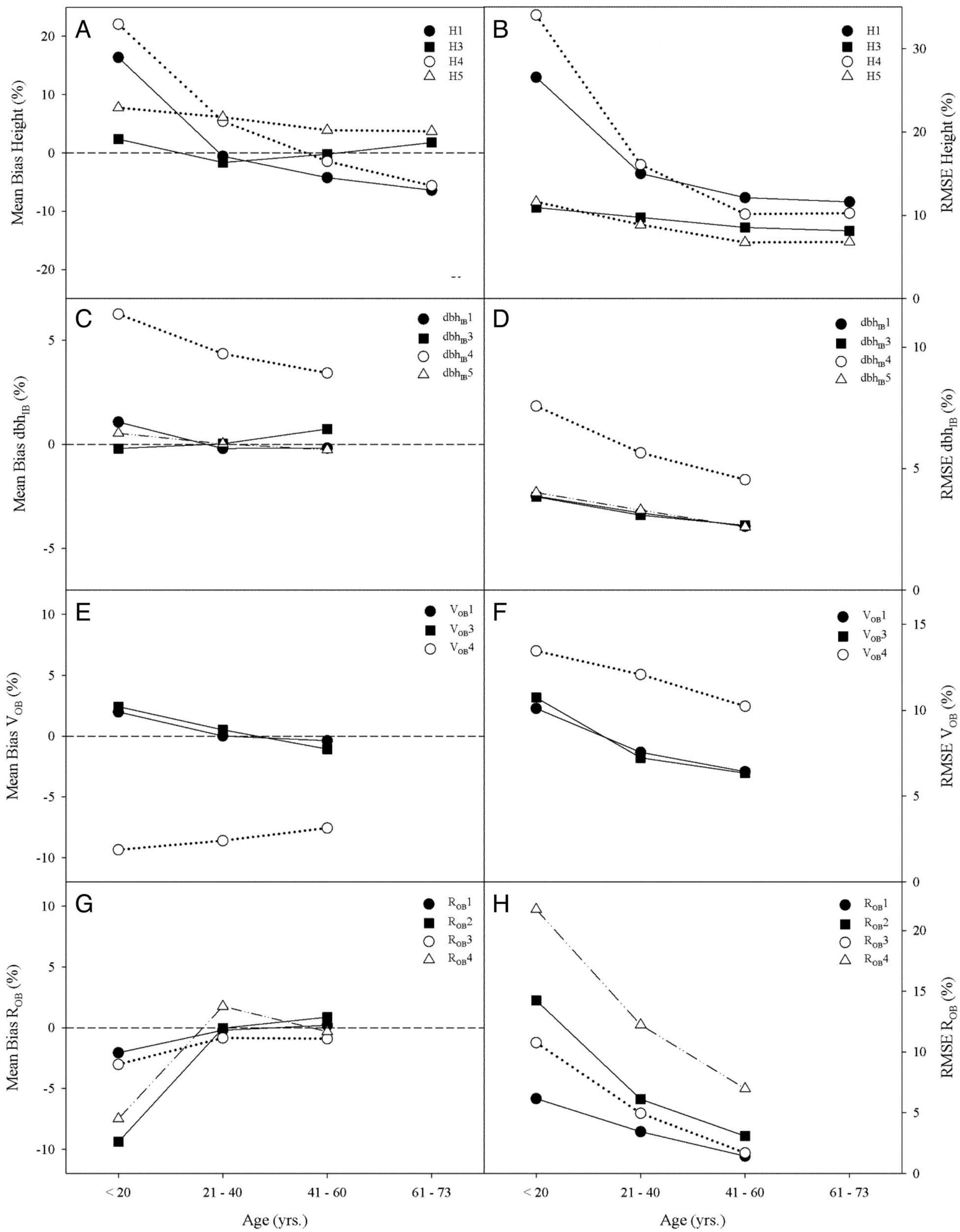

Figure 6. Mean Bias (A, C, E, and G) and RMSE (B, D, F, and H) presented as percentages of the mean of the models reported in this study and in the literature to predict total H (A and B), dbh_{IB} (C and D), V_{OB} (E and F), and stem volume ratio outside bark (R_{OB}) (G and H) of longleaf pine trees across four stand age classes: <20, 21–40, 41–60, and 61–73 years. The H models are current report general model (H1), current report local model (H3), Quicke and Meldahl (1992) (H4), and Leduc and Goelz (2010) (H5). The dbh_{IB} models are current report general model ($dbh_{IB}1$), current report local model ($dbh_{IB}3$), Farrar (1987) ($dbh_{IB}4$, assuming a live crown ratio <36%), and Farrar (1987) ($D_{IB}5$, assuming a live crown ratio >50%). For V_{OB} the models are current report general model ($V_{OB}1$), current report local model ($V_{OB}3$), and Baldwin and Saucier (1983) ($V_{OB}4$). For R_{OB}, the models are current report using d_t = 10.16 cm ($R_{OB}1$), current report using d_t = 15.24 cm ($R_{OB}2$), Saucier et al. (1981) using d_t = 10.16 cm ($R_{OB}3$), and Saucier et al. (1981) using d_t = 15.24 cm ($R_{OB}4$).

for naturally regenerated longleaf pine trees, only use dbh as an explanatory variable) and Leduc and Goelz (2010) (H5, for planted longleaf pine trees, use dbh, H_{dom}, and quadratic mean diameter as explanatory variables) had Bias of approximately 22.1 and 7.7%, respectively. On the other hand, the model reported in this study that included stand parameters (H3, uses dbh, N, BA, and SI as explanatory variables) had a Bias of 2.4%. In the age class 41–60 years, all models showed Bias ranging between −4.2 and −0.2%. For the age class 61–73 years, the model reported by Leduc and Goelz (2010) had smaller RMSE than all other models. Across all age classes, H3 had smaller Bias than all other models (Figure 6A).

For dbh_{IB} estimations, the models reported in this study, across all age classes, ranged in Bias between −0.2 and 1.1% ($dbh_{IB}1$) and −0.2 and 0.7% ($dbh_{IB}3$), whereas the model reported by Farrar (1987) predicted well if a live crown ratio[1] larger than 50% was assumed with Bias ranging between −0.3 and 0.5%. If a live crown ratio lower than 36% was assumed, the Bias of the model of Farrar (1987) increased, ranging between 2.1 and 3.4%.

For V_{OB}, across age classes, the model reported by Saucier et al. (1981) underestimated by approximately 8.5% and had an average RMSE of 11.9%, whereas the model reported in this study that used dbh and H ($V_{OB}1$ and $V_{OB}3$) had a Bias and RMSE between −1.1 and 2.4% and 6.3 and 10.7%, respectively. For merchantable volume ratio outside bark, the models tended to underestimate for trees younger than 20 years old (Figure 6G). For example, for $d_t = 10.16$ cm, the model reported in this study and the model reported by Baldwin and Saucier (1983) had a Bias of approximately −2.1% ($R_{OB}1$) and −3.0% ($R_{OB}3$), respectively. For $d_t = 15.24$ cm, the Bias was approximately −9.4% ($R_{OB}2$) and −7.5% ($R_{OB}4$), respectively, but for older trees the Bias was drastically reduced, being always lower than 1.9% (Figure 6G). For R_{OB} estimations, RMSE of the model reported in this study were always smaller than the model of Baldwin and Saucier (1983) (Figure 6H). Responses similar to V_{OB} and R_{OB} were observed for inside bark estimations (data not shown).

Model Validation Using External Data

When models to estimate H and V_{OB} were evaluated using trees measured outside the geographical range of the model development data set (Fort Benning, Georgia), across stand age, there was no difference between observed and predicted values for H and V_{OB} for any of the predicting models reported ($P > 0.11$). For H estimations, the model that only depended on dbh (local model H1, Figure 7A) showed a tendency to overestimate H (only in 87-year-old stands the model underestimated by 7%), with errors ranging between 13 and 30% underestimations. When stand parameters Age and BA were included in the general model H2, the relationship between observed and predicted values, across all ages, was improved compared with model H1 (Figure 7C), but there were still significant differences between observed and predicted values (those differences ranged between 17% underestimations for 5-year-old stands to 1.8% overestimations for 87-year-old stands). When SI was incorporated into the model (general model H3), there were no differences between observed and predicted values for any stand (age; Figure 7E). Similar results were observed for residual distribution (Figure 7B, D, and F). For all models, larger differences were found at the 64-year-old naturally regenerated stand, perhaps because of its low productivity (average SI of 19.6 m), lower than the minimum values observed in fitting data set (SI for other stands ranged between 22.4 and 33.9 m). Because V_{OB} was measured only on 11 trees at Fort Benning, no model validation within stand age was performed and as was stated previously, there was no difference between observed and predicted values for any of the models to predict V_{OB}.

Discussion

The set of prediction equations for longleaf pine trees reported in this study provide useful tools for the study and management of this species. General and local models are presented for H, dbh_{IB}, V_{OB}, and V_{IB} estimations. The user should decide which model to use, depending on data availability and level of accuracy desired.

The model selected for H estimations was compared against linear and nonlinear equations reported elsewhere (Curtis 1967, Arabatzis and Burkhart 1992, Huang et al. 1992, Staudhammer and LeMay 2000, Temesgen et al. 2007, Leduc and Goelz 2010, Bi et al. 2012). The final model selected showed predictive ability similar to that of models in the literature (data not shown), but at the same time allowed the incorporation of stand-level parameters selected by a statistical procedure that included VIF discrimination. Similar to our study, other authors such as Staudhammer and LeMay (2000), Temesgen et al. (2007) and Leduc and Goelz (2010) also included stand parameters in their final models, concluding that measures of stand density and development should be included in dbh-H models to improve accuracy of H predictions. We also provide the option to include measures of stand productivity (i.e., SI), which produced more accurate predictions. Leduc and Goelz (2010) used a similar approach by including H_{dom} in their model to estimate H. These authors also included quadratic mean diameter in their model, a direct combination of N and BA. In our case, the inclusion of stand-level parameters highly improved the accuracy of the model, essentially eliminating bias for trees with H larger than about 25 m.

The equation for dbh_{IB} and, hence, the estimations of bark thickness (as bark thickness can be determined as the difference between dbh and dbh_{IB}) showed little improvement with the inclusion of stand-level parameters. Similar results were reported for *Pinus taeda* (loblolly pine) trees, for which bark thickness was linearly correlated with dbh but not associated with stand density (Feduccia and Mann 1975) or stand age (Johnson and Wood 1987). In addition, for loblolly pine trees, Tiarks and Haywood (1992) reported no effect of weed control and fertilization on the relationship between bark thickness and dbh. For longleaf pine, Farrar (1987) also reported equations that linearly correlated dbh_{IB} with dbh for different live crown ratio classes. Those equations performed well but relied on additional measurements (total tree height and height up to live crown base to estimate live crown ratio) that are not always available. Furthermore, the reported model performed better than the models presented in this study when live crown ratio was assumed to be larger than 50%, a value that is generally associated with smaller trees (Farrar 1987).

Stem volume is a key parameter for forest owners, managers, and researchers. Different types of analyses, such as economics, restoration ecology planning, or carbon sequestration accounting, are directly or indirectly dependent on estimation of stem growth, so equations for accurate determinations of V are critical. We present a set of equations for both outside and inside bark V estimations that can be improved if H measurements are available. The best model used dbh and H as the independent variables and showed almost no improvement if stand-level parameters were included. On the other hand, the model that used only dbh as an explanatory predictor was

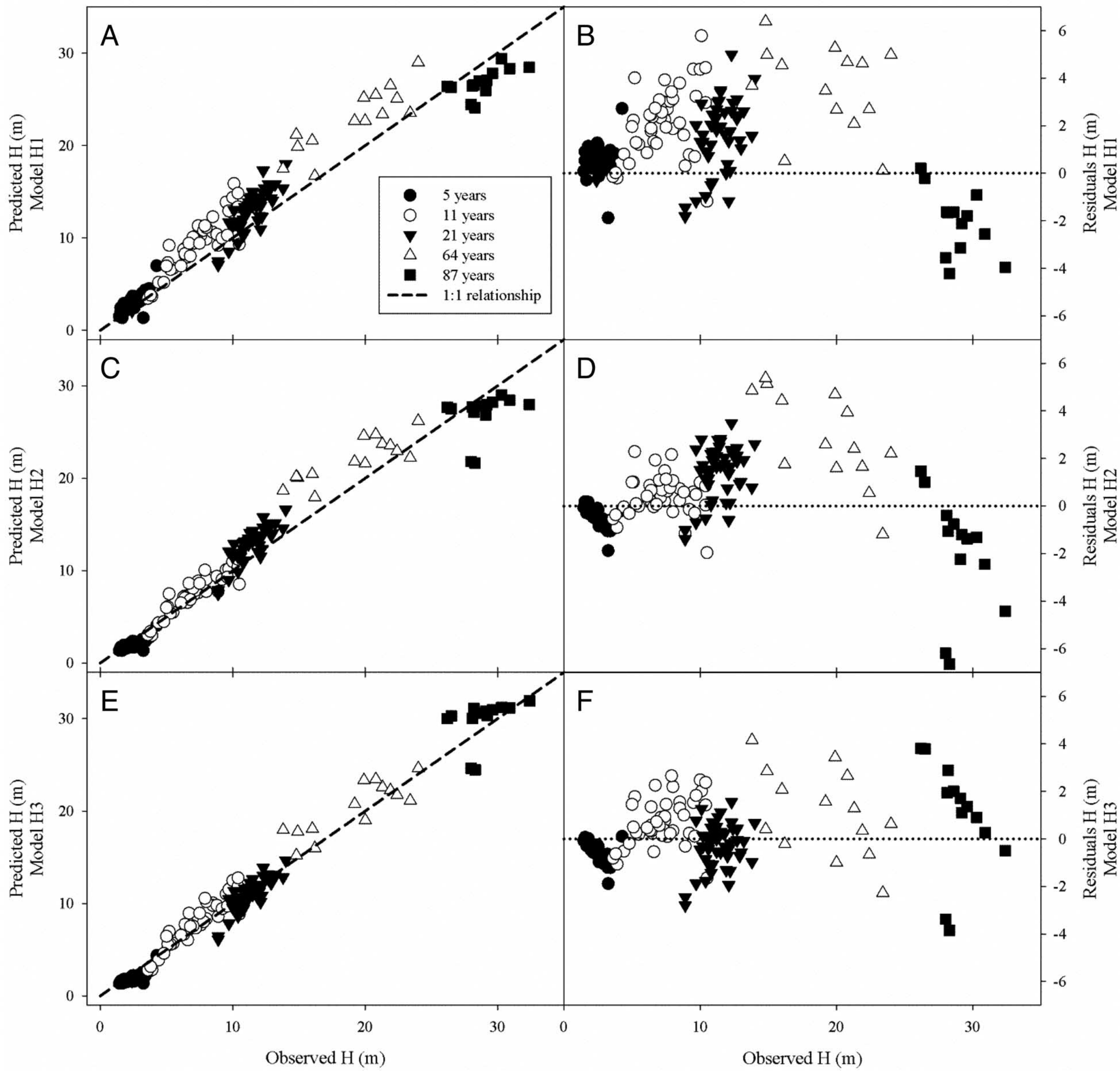

Figure 7. Evaluation of total tree height (H) models for stands of different ages at Fort Benning, Georgia. Observed versus predicted (simulated) values (A, C, and E) and residuals (predicted − observed) versus observed values (B, D, and F) of H using local model H1 (A and B), general model H2 (C and D), and general model H3 (E and F).

improved when Age and BA were included, reducing Bias to the same magnitude as the model with dbh and H. This result implies that differences in stem tapering that affect the relationship between dbh and H can be successfully addressed by the inclusion of stand parameters without the need for direct measurements of H.

Another option to determine V when H is unknown is to estimate H using a dbh-H equation and then use the estimated H into one of the functions that use dbh and H to determine V. When we tested this approach, better results were obtained based on models that relied only on dbh and stand parameters (data not shown). Because of that, in the case of H unknown, we recommend use of models V2 or V3 instead of models V4 to V6 with estimated H.

It is important to note that the parameter value of the power of dbh (c_2, d_2) in Equations 5–12 had a value slightly different from 2, a value generally used to correlate dbh with V (Baldwin and Saucier 1983, Van Deusen et al. 1981). Interestingly, the value of parameters c_2 was reduced from approximately 2.188 ($V_{OB}1$) to 2.099 ($V_{OB}2$) and 2.078 ($V_{OB}3$) when stand-level parameters were incorporated into the equation. When H was included into the model, instead of using $dbh^2 \cdot H$, we determined that the power of dbh and H, instead of 2 and 1, should be 1.917 and 1.072, respectively, for the local model $V_{OB}4$ and 1.853 (dbh) and 1.012 (H) and 1.851 (dbh) and 1.047 (H), for the general models $V_{OB}5$ and $V_{OB}6$, respectively. These results imply that models relying on dbh^2 or $dbh^2 \cdot H$ should be revised, and future models for this species should determine the correct power of dbh and H.

The approach presented by Burkhart (1977) to estimate stem volume ratio to any top diameter was adequate for our data set. Similar to the model of Saucier et al. (1981), when the equations presented in this study were tested for trees younger than 20 years (top limit diameter close to dbh), results showed a tendency to underestimate merchantable volume ratio. Nevertheless, for trees older than 20 years, the bias was negligible. Hence, the equations presented in this study are a valuable tool for foresters who need to estimate merchantable volume to any stem diameter limit.

The models reported in this study performed well for the data set used for evaluation. When the equations to estimate H and V_{OB} were tested in a data set obtained in stands located outside the geographical zone of the data used for model development (Fort Benning, Georgia), the results support the robustness of the models. When stand parameters were included in the models, there were no differences in observed and predicted values for H for any stand age tested, even in naturally regenerated stands (64- and 87-year-old stands). Because of cost constraints, our validation of V_{OB} with data from Fort Benning was performed on 11 trees with ages ranging between 21 and 87 years. This short data set does not allow us to properly validate V_{OB} within stand ages, but, in any case, across stand ages there were no differences between observed and predicted V_{OB}. This result suggests that the models are a robust alternative for H and V_{OB} estimations on planted stands (and perhaps naturally regenerated stands as well) across a wide range of ages. Further validation will be carried out in the future using data from stands located in the Kisatchie National Forest (Louisiana) and Camp Lejeune (South Carolina).

Even though we strongly recommend using the equations within the range of data used to fit (see Table 1), the results presented in this validation study provide a valuable alternative to available models and are intended as a tool to support present and future longleaf pine management decisions.

Endnote

1. Live crown ratio is defined as 100 • length of full live crown/total height of the tree.

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